Market-specific human capital: Talent mobility, compensation, and shareholder value

Neil Brisley*†‡

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Abstract

Country-specific and industry-specific human capital are potentially important factors in the market for talent. In a competitive matching model, we analyze how the distribution and composition of human capital affects how the top firms and managers from one market compete with those from another, to develop an integrated market for talent. When talent has general and market-specific human capital (GHC and M-SHC) components, the matching problem becomes more complex. We derive the unique integrated market equilibrium to show how talent 'migration' (or cross-market hiring) may increase or decrease overall average productivity, compensation, and shareholder value – compared to a constrained value-maximizing benchmark, the market equilibrium leads to 'too much' cross-market hiring. We also identify circumstances where i) no migration will occur, despite the absence of any other barrier to integration; ii) a 'brain drain' occurs, despite the migrating managers creating less value in the destination market than at home; and iii) migration is non-monotonic in M-SHC.

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†Associate Professor of Finance, School of Accounting & Finance, University of Waterloo, Ontario, Canada.
https://uwaterloo.ca/school-of-accounting-and-finance/people-profiles/neil-brisley
Email: nbrisley@uwaterloo.ca

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1 Introduction.

In the market for top executives, general management skills are becoming more important than firm-specific human capital, and talent is increasingly mobile (e.g., Custódio, Ferreira, and Matos, 2013; Murphy and Zábojník, 2004, 2007; Frydman, 2015). However, significant differences persist in hiring patterns and compensation norms across continents, countries, cultures, sectors, industries, and firms. Even in the absence of formal barriers to mobility, industry-specific and country-specific human capital are each potentially important factors and constraints on how (or even *whether*) an integrated market for talent may develop. While 30% of the U.K. FTSE100 CEOs joined from a role in a different industry,[1] Cremers and Grinstein (2014) demonstrate the importance of heterogeneous industry-specific skills and find external CEO hiring is more common in some U.S. sectors than others. Only 14% of the 2013 Fortune Global 500 CEOs come from countries other than their corporate homelands, and the biographies of ten profiled Fortune 500 CEOs show that each obtained significant experience in their adoptive country before attaining that position.[2]

As emphasized in Murphy and Zábojník (2004), *"market forces and the composition of managerial skills are of first-order importance in determining the trends in CEO pay and turnover."* So in response to a shift in the competitive landscape, or to a change in the potential impact, scope or applicability of different managerial skills, what are the effects on firms' incentives to hire top talent from other markets, and on managers' ability to switch industry or country? Who would win or lose? What is the role of 'market-specific human capital' (M-SHC) – the part of a manager's talent that *cannot* be utilized outside of a given 'home' market – versus her general human capital (GHC) which can also be applied in 'away' markets?

In a competitive matching model, and in each of two heterogeneous distinct segregated markets, we decompose talent into M-SHC and GHC. We then *"Tear down this Wall!"* between the two markets to analyze how their largest firms and top managers would compete in a single (e.g., inter-industry, or cross-border) integrated market for talent. We contribute the unique integrated market equilibrium to show how self-interested 'migration' by managers (i.e., rational 'cross-market' hiring

[1] Robert Half, 2015 CEO Tracker: www.roberthalf.co.uk/ftse-ceo-infographic
[2] Fortune.com/2013/07/08/10-Global-500-CEO-strangers-in-strange-lands

by firms) may increase *or decrease* overall average productivity, compensation, and shareholder value, depending on the gains and losses from rematching, the redundancy of M-SHC when hired away, and the wage impact of the revised competitive outcomes. Compared to a constrained value-maximizing benchmark, the market equilibrium leads to 'too much' cross-market hiring. We also identify circumstances in which i) no migration will occur, despite the absence of any other barrier to integration; ii) a 'brain drain' occurs, despite the migrating managers creating less value in the destination market than at home; and iii) greater M-SHC *increases* the number of managers migrating, *and* their compensation, despite reducing their productivity in the destination market.[3]

In our model, managers are distributed and ranked by talent; firms are distributed and ranked by size (more specifically, *value-sensitivity* to manager talent), both initially à la Gabaix and Landier (2008; henceforth 'GL'). GL derive the wage in a segregated market where their one-dimensional manager talent leads to an unambiguous ranking by hiring firms, and value-complementarity of talent and firm size leads to positive assortative matching (PAM) – the largest firm matches with the most talented manager and the n-ranked firm matches with the n-ranked manager. To motivate the potential for cross-market hiring, we first show that firm size distribution *relative* to total human capital density determines which of our two segregated markets is 'talent rich', and has lower wages for same-sized firms than the 'talent poor' (i.e., 'opportunity-rich') market. For example, we find markets with *more* larger firms may pay cross-sectionally *lower* wages, specifically if they have a surfeit of talented managers relative to firm size.[4]

Next, we decompose manager talent into M-SHC and GHC components, then 'desegregate' ('unify', or 'pool') the two markets to permit cross-market hiring, but any manager hired 'away' cannot use her M-SHC. Firms from different markets will rank identical managers differently, so PAM cannot be applied. This represents a sophistication of the matching problem, to which we contribute the unique equilibrium.

Integration is characterized as some managers from the talent-rich market 'migrate' – are hired away at an increased wage by firms in the opportunity-rich market. This exodus decreases the competition faced by their stay-at-home former peers, who see *their* wages also increase, as they are

[3] We contribute results from combining multiple independent firm size distributions following a Zipf Law, and multiple independent talent distributions appealing to Extreme Value Theory.

[4] Jung and Subramanian (2015) focus on product market competition in assumed segregated industries and talent pools, and estimate "significant inter-industry variation in the inferred distributions of firm quality and CEO talent".

promoted to larger firms at home. Those firms suffer from paying a higher wage and to lesser talent than before, so reducing firm value on a gross (before-wage) and net (after-wage) basis. Opposite (but not equal) effects pertain in the destination talent-importing, opportunity-rich market, whose original managers suffer increased competition, relegation to smaller firms, and reduced wages; and whose firms enjoy recruiting improved talent at a lower wage than before, so increasing firm value.

Cross-market hiring continues – and the gap between the wages at same-sized firms narrows – until the wage available to a manager is the same away as at home, taking into account the redundancy of her M-SHC if she does migrate away, and the size of firm that will hire her. The coexistence of two parallel wage functions in the integrated market equilibrium implies cross-sectional variation in wages at same-sized firms, *even* when talent migration is freely permitted and significant. The opportunity-rich market maintains more of its proportional wage premium post-unification, the more of the human capital from the talent-rich market is market-specific.

In the special case of 100%-GHC, a single equilibrium wage function applies across the unified market, so that managers of the same talent earn the same wage at a firm of the same size, *whatever* the origin of firm or manager. Firms from the opportunity-rich market gain more from unification than firms from the talent-rich market lose – we find average net *gains* to shareholders overall, thanks both to more productive matching and to a reduction in the average wage paid by same-sized firms. Managers from the talent-rich market gain from unification exactly what managers from the opportunity-rich market lose – the net wage effects on equally-talented managers are a zero-sum game, when talent is all GHC.

Our most novel results are driven by the redundant M-SHC of managers hired cross-market. The individual winners and losers from unification are the same as in the 100%-GHC case, but the *value* they win or lose varies with our market-wide M-SHC measure, so the overall averages (across same-sized firms, in the unified market) of productivity, wages paid, and shareholder value, may each increase *or* decrease on unification, and each are non-monotonic in the level of M-SHC. Average wage paid attains an internal maximum in M-SHC; productivity and shareholder value attain internal minima. *If M-SHC is sufficiently high, then there is destruction of overall weighted average net shareholder value at same-sized firms across the unified market; shareholders are worse off with unification than without.* This happens when the productivity losses in the talent-exporting market outweigh the gains in the talent-importing market, due to the redundancy of M-SHC on migration, which also increases wages paid in the former talent-rich market more than it decreases them in the opportunity-rich market. Relative to the 100%-GHC unification case, firms are worse off, on average, with M-SHC. Yet average wages earned by equally-talented managers *increase* on unification, are non-monotonic in M-SHC, and attain an internal maximum.

The market equilibrium is characterized by a constant fraction of managers at each ranking in the talent-rich market being hired away, and this profile arises endogenously in the model. The fraction migrating increases in the relative talent-to-size disparity between the two markets. It also represents *'too much' migration*, compared to a constrained out-of-equilibrium value-maximizing constant fraction. This is because the initial wage disparity between markets creates talent arbitrage opportunities for firms in the opportunity-rich market (i.e., wage arbitrage opportunities for managers in the talent-rich market), until equilibrium is reached *beyond* the point where further migration begins to reduce value overall.

Although managers hired away certainly increase their own wages, they may all become employed more *or less* productively than before. This depends on M-SHC directly since it reduces their effective talent, and indirectly as it determines the destination firm size they match with, which may increase *or decrease*. Moreover, cross-market hiring can occur from the talent-rich market *even* if its managers have GHC (relative to firm-size) *lower* than the total human capital (relative to firm-size) of managers in the opportunity-rich market. These 'brain drain' contributions suggest empirically that a talent-rich (opportunity-poor) market with high M-SHC may experience a depletion of its talent resources, even though their exportable GHC is comparatively low, and yields only limited benefits to firms in the destination market.

Above a threshold level of M-SHC, and even when there are no other barriers to talent mobility, we find that no manager will be hired away; even though not formally segregated, the two markets fail to integrate because of the prevalence of M-SHC.[5] Our model therefore also encompasses 'organic integration' scenarios in which, for example, Industry-SHC is initially above the level which *precludes* integration, but then some shock or evolution leaves it below the threshold and stimulates the desegregation of once-distinct talent pools. For example, Custódio, Ferreira, and Matos (2013) build on Murphy and Zábojník (2004, 2007) and Frydman (2015), to provide direct evidence

[5] The European Union enshrines free movement of labor between its 28 member states. However, 'foreign' CEOs are still a small minority, and usually originate from culturally, linguistically or geographically adjacent countries.

of the *"increased importance of general managerial skills over firm-specific human capital in the market for CEOs in the last decades"*. Further, some formerly market-specific skills could *become* GHC and so make their proprietors more attractive for cross-market hiring.[6] Finally, the 'no integration' threshold for M-SHC is itself increasing in the relative size (value-sensitivity to talent) of the opportunity-rich market, and in the relative total talent of the talent-rich market, so that upward shocks to either may initiate integration.[7]

If one market is sufficiently talent rich compared to the other, then, relative to the 100%-GHC case, we find an increasing proportion of M-SHC in that market initially causes *increased* migration; even though it handicaps the effective talent of migrating managers, it increases their wage gains. This result stems from the dual determinants of a manager's wage: first, her talent determines her ranking (which helps); second, the *density* of talent she competes against (which hurts). More M-SHC (i.e., less GHC) can *benefit* these managers by softening the wage-reducing competition they collectively induce when they migrate to an opportunity-rich market; they achieve a better wage than if M-SHC was lower and they matched at a higher-ranked firm but against stiffer competition. Eventually, the migrating fraction *is* decreasing in M-SHC, and none will migrate once M-SHC exceeds a threshold. Hence, in a sufficiently talent-rich market, there exists a positive level of M-SHC which maximizes their managers' wage gains on unification, maximizes the fraction migrating, but maximizes the losses to gross and net firm value.

This result contributes a subtle contrast to the classic 'hold-up' problem (Becker, 1962), wherein Firm-SHC decreases a manager's bargaining power by reducing her reservation wage from outside options. Similarly, any *individual* manager would prefer that all her human capital be GHC, rather than M-SHC. Differently, we identify circumstances where *all* top managers in the talent-rich market gain more from unification when some of their human capital is market-specific, rather than if it were all GHC. Paradoxically, M-SHC *increases* their outside wage option, by committing them not to compete too strongly for those opportunities.[8]

[6] 'Tech' skills, once applicable only in specialized industries, are increasingly in demand across the mainstream. Also, Surovtseva (2014) finds that higher-skilled second-generation Chinese or Mexican residents in the U.S. experienced a positive shock to their income when China joined the WTO or when Mexico signed NAFTA – these residents' endowment of underutilized Country-SHC suddenly finding a more general application.

[7] Célérier and Vallée (2015) document that finance industry returns to talent have increased over past decades, to 3-times those in the rest of the economy.

[8] In Frydman (2015), within a single standalone market and in the presence of Firm-SHC, as GHC becomes more valuable so managers' outside options and mobility increase, leading to higher pay.

Our paper is firmly in the paradigm of competitive markets determining CEO pay (Rosen, 1982; Himmelberg and Hubbard, 2000; Murphy and Zábojník, 2004, 2007; Gabaix and Landier, 2008). We build on the GL model, which offers intuitively appealing distributional assumptions that are theoretically sound, empirically supported, and contribute tractability to the tradition of matching models (Lucas, 1978; Rosen, 1981, 1982, 1992; Sattinger, 1993; Teulings, 1995; and Terviö, 2008). Competitive matching models and PAM find empirical support in Eisfeldt and Kuhnen (2013);[9] Nguyen and Nielsen (2014); Falato, Li and Milbourn (2015); and Pan (2016).

Our 'cross-market' model could apply to national boundaries,[10] or to industry sectors.[11] The identity of the winning and losing constituencies in desegregation is not surprising in our model. However, when M-SHC is significant, our results on *overall* average value destruction indicate a potential 'dark side' to top talent mobility, which may counsel caution on regulators seeking to unify and liberalize talent markets.[12] Moreover, at *all* levels of M-SHC and from the weighted average perspective of same-sized firms, our results indicate that the market equilibrium may involve 'too much' cross-market hiring, unless other unmodelled frictions are at play. These observations may also contribute to the debate over brain drain towards the finance sector, talent competition and wage premiums paid therein (e.g., Philippon and Reshef, 2012; Célérier and Vallée, 2015; Acharya, Pagano, and Volpin, 2016; Böhm, Metzger, and Strömberg, 2016). Finally, our model could apply to talent hierarchies 'cross-division' at the individual firm level. Consistent with our results, Tate and Yang (2015a; 2015b) find diversifying mergers occur more frequently and more durably between industry pairs with higher transferability of human capital, permitting internal labor-market productivity gains – either immediately upon unification of the firms, or in response to subsequent industry shocks.

[9] Their theoretical model focuses on Firm-SHC, which ensures firing and replacement occurs only for managers below a minimum talent threshold.

[10] The GL empirical design assumes segregated *national* CEO markets while acknowledging that they may be becoming more integrated over time – they exclude only Belgium, which they find fairly integrated with France and the Netherlands. U.S. CEOs earned almost triple the compensation of their matched UK counterparts in 1997 (Conyon and Murphy, 2000), but U.S. and non-U.S. pay converges in the 2000's (Fernandes, Ferreira, Matos, and Murphy, 2013). On the other hand, recent evidence also suggests that even the *intra*-U.S. CEO market is still geographically segmented (Bouwman, 2013; Yonker, 2015; Zhao, 2014; Broman, Nandy, and Tian, 2016).

[11] For Industry-SHC see, for example, Neal (1995); Parent (2000); Guren, Hémous, and Olsen (2015). Cremers and Grinstein (2014) build on Parrino (1997), who argues that even when CEOs come from a different industry, they often have some industry-relevant experience, either because they worked in the industry in the past or because their present firm operates in more than one industry.

[12] Ours is not a full welfare analysis, and our results may be most applicable to the largest firms and the most talented managers. However, these are often the ones empiricists study, and those with the most significant lobbying influence on regulatory policy.

2 Model: Segregated talent markets.

We briefly reproduce a simplified version of the GL headline result as Lemma 1.

2.1 Gabaix and Landier (2008).

Consider a continuum of the largest firms $n \in [0, N]$, and a continuum of the most talented managers $m \in [0, N]$. Firms have size

$$S(n) = \frac{A}{n} \tag{1}$$

for some $A > 0$, a 'Zipf Law' distribution.[13]

Managers have talent $T(m)$, where (appealing to Extreme Value Theory)[14]

$$T'(m) = -Bm^{-(1-\beta)} \tag{2}$$

for some $\beta < 1$ and $B > 0$.

There is a measure n of managers, and of firms, in the interval $[0, n]$ so that n can be understood as the rank, or quantile of rank. A manager's talent acts multiplicatively with firm size to create *gross* value (i.e., before wages) of ST for the firm they lead, so that the most talented managers have the highest value impact on the largest firms (e.g., Rosen, 1992). More generally, S is the firm's *value-sensitivity* to manager talent.[15] Firms take the wage of each manager as given and compete to hire from the talent pool, the n-ranked firm having objective

$$\max_{m} \qquad S(n)\,T(m) - w(T(m)), \tag{3}$$

where wage $w(T)$ gives the market wage of a manager with talent $T = T(m)$. The function $w(T)$ is determined endogenously in a competitive equilibrium in which one manager is allocated to each

[13] More generally, this is a Pareto distribution $S(n) = An^{-\alpha}$. GL find that $\alpha = 1$ fits firm size data well. Other phenomena (e.g., city size) also closely follow Zipf's Law. For a review, see Gabaix (2009).

[14] For manager talent originating from regular continuous probability distributions, extreme value theory implies that the spacings of observations in the *upper tail* (i.e., for 'small' m, e.g., the Top 1,000 managers in a population of a million) of the distribution are related as in (2) (either exactly, or up to a 'slowly varying' function), where $-\beta$ is the 'tail index' of the distribution of talents.

[15] GL allow for gross value $CS^{\gamma}T$. They find constant returns to scale, $\gamma \approx 1$, supported empirically. If C varies across firms, then talent matches with firms ordered by 'effective firm size', CS, rather than by absolute firm size, S, and their analysis proceeds as here. We omit C for clarity.

firm, and the manager m is competitively assigned to firm n. The assignment function is $m = M(n)$. Each firm chooses its manager optimally, $M(n) \in \arg\max_m \quad S(n)T(m) - w(T(m))$.

The first order condition from objective (3) is $w'(m) = S(n)T'(m)$, so the marginal wage cost to the firm of a slightly better manager equals its marginal benefit. Due to the value-complementarity between firm size and manager talent, any equilibrium exhibits PAM, whereby the n-ranked firm hires the n-ranked manager (i.e., $n = m = M(n)$), so $w'(n) = S(n)T'(n) = -ABn^{\beta-2}$ and we have:

Lemma 1 . ***(Gabaix and Landier, 2008). Wages in the market equilibrium.***

The n-ranked manager of talent $T(n)$ runs the n-ranked firm of size $S(n)$. Let n_* denote the rank of some reference firm (e.g., $n_* = 250$), then considering the domain of the larger firms (small n) let N increase so that $\frac{n}{N} \to 0$ (i.e., $n << N$). In the limit the equilibrium wage is

$$\begin{aligned} w(n) &= \frac{AB}{(1-\beta)} n^{-(1-\beta)} &(4)\\ &= D(n_*)\, S(n_*)^{\beta}\, S(n)^{(1-\beta)} &(5) \end{aligned}$$

where $D(n_*) = \frac{-n_* T'(n_*)}{1-\beta}$ and $T'(n_*) = -B(n_*)^{-(1-\beta)}$, and $w(n_*) = D(n_*)\, S(n_*)$.■

All proofs of Lemmas and Propositions are in Appendix A.

Under GL's additional assumption that talent pools in parallel segmented countries are *identically distributed,* formulation (5) implies wages at same-sized firms across segmented countries vary with the size $S(n_*)$ of the reference firm in that country.[16] GL explain the higher pay of U.S. firms compared to, say, German firms of the same size, arguing that the concentration of larger firms in the U.S. bid more for the relatively scarce talent of the executives in that market. In the present paper we focus on formulation (4).[17]

[16] GL term this the *Dual Scaling Equation.* In a single closed market it has a cross-sectional interpretation; wage is proportional to firm size to the power $(1-\beta)$, a result they dedicate as 'Roberts' Law' (Roberts, 1956), and they estimate empirically $\beta \approx \frac{2}{3}$.

[17] GL draw the time series implication from (4) in a single closed market, of a linear relation between wage and firm size; when all firm sizes double $(A \to 2A)$, so wages double. Indeed, GL relate the six-fold increase in U.S. CEO pay from 1980-2003 to the six-fold increase in size of the largest U.S. firms over that period.

2.2 Two segregated markets, different firm-size and talent distributions.

Consider two markets, M1 and M2. Each has its own distribution of firm size and manager talent, with parameters A_i, B_i, characterizing the functions $S_i(n)$, $T_i'(m)$. Lemma 1 gives wages $w_i(n) = \frac{A_i B_i}{(1-\beta)} n^{-(1-\beta)}, \quad i \in \{1, 2\}$.

Define $\lambda = \frac{A_2}{A_1}$, i.e., $A_2 = \lambda A_1$ for some $\lambda > 0$, then λ is a measure of firm size (i.e., *value-sensitivity to talent*) in M2 *relative* to M1. Distributions $S_i(\cdot)$ are related by a scaling of rankings; $S_1(m) = S_2(\lambda m)$, the m-ranked firm in M1 is the same size as the λm-ranked firm in M2.

We integrate the Extreme Value Theory relation (2) and put $T_i(m) = K - B_i \frac{m^\beta}{\beta}$ so that either market has the same theoretical upper bound K on managerial talent, but the differing coefficients B_i measure *absolute talent scarcity* – how steeply talent declines as we dig deeper into the talent pool in that market. Define $\phi = \left(\frac{B_1}{B_2}\right)^{\frac{1}{\beta}}$, i.e., $B_1 = B_2 \phi^\beta$ for some $\phi > 0$, then ϕ is a measure of the spacing of talent in M1 *relative* to M2. Talent distributions $T_i(\cdot)$ are related by a scaling of rankings; $T_1(m) = T_2(\phi m)$, the m-ranked manager in M1 has the same total human capital as the ϕm-ranked manager in M2.

Lemma 2 . ***Segregated markets. Comparing same-sized firms, and equally-talented managers.***

Consider the n-ranked M1 firm, size $S_1(n)$, hiring the n-ranked M1 manager with talent $T_1(n)$ at wage $w_1(n)$;

i) Fixing firm size, the λn-ranked M2 firm, size $S_2(\lambda n) = S_1(n)$, hires the λn-ranked M2 manager with talent $T_2(\lambda n) = T_1(n) - \left(\lambda^\beta - \phi^\beta\right) \frac{B_2}{\beta} n^\beta$, at wage $w_2(\lambda n) = \left(\frac{\lambda}{\phi}\right)^\beta w_1(n)$;

Firms in M1 employ managers of higher talent and at lower wage than do same-sized firms in M2, if and only if $\phi < \lambda$.

ii) Fixing manager talent, the ϕn-ranked M2 firm, size $S_2(\phi n) = \frac{\lambda}{\phi} S_1(n)$, hires the ϕn-ranked M2 manager with talent $T_2(\phi n) = T_1(n)$, at wage $w_2(\phi n) = \frac{\lambda}{\phi} w_1(n)$;

Managers in M1 earn less and lead a smaller firm than do equally-talented managers in M2, if and only if $\phi < \lambda$.■

To maintain generality, we placed no prior assumption on *which* of the two markets has the absolute stronger manager talent distribution. For example, if $\phi < 1$ then M1 has nominally more managers exceeding any given talent level than does M2. However, part (i) emphasizes that 'what matters' is the distribution of talent in either market *relative* to the distribution of large firm-size opportunities in that market: *If and only if* $\phi < \lambda$ does M1 have managers of higher talent *for same-sized firms* than does M2. Then and only then does the cross-sectional wage at a same-sized firm in M2 exceed that in M1, due to the relative scarcity of talent in M2.

Without loss of generality, *for the remainder of the paper we focus on* $\phi < \lambda$ and *describe M1 as 'talent-rich' compared to M2, which is 'talent-poor', or 'opportunity-rich'.*[18] We remain general as to which of the two markets has larger firm size: if $\lambda < 1$ ($\lambda > 1$) then M1 is the larger (smaller) market.

The GL conclusion on cross-sectional wage comparisons follows from their assumption that $B_1 \equiv B_2$ while $A_1 < A_2$ (i.e., $\phi = 1$; $\lambda > 1$). Our freedom to analyze more general $\phi > 0$ permits the empirical implication that segregated markets with *more* large firms may nevertheless pay cross-sectionally *lower* wages, specifically if the market is talent-rich.[19] More importantly, our $\phi > 0$ contributes further breadth to our analysis of what happens when two standalone markets are unified, in the presence of M-SHC, to permit cross-market talent mobility, an analysis outside the scope of GL.

3 Analysis: Unifying two segregated markets, with M-SHC.

We now desegregate the two markets, permitting firms from either market to compete to hire managers from the unified talent pool. In the previous section, the only relevant parameter for any manager was her talent in her home market. Now, with the prospect of cross-market hiring, we contemplate an M-SHC portion of the manager's talent which cannot be productively employed outside of her home market. Of course, the remainder of her talent consists of GHC which, combined with ex ante wage disparities in the segregated markets, may be sufficient to make managers from a talent-rich market attractive for cross-market hiring.[20]

[18] In fact, $\phi < \lambda$ is equivalent to $B_1 < \left(\frac{A_1}{A_2}\right)^{\beta} B_2$.

[19] Part (ii) also offers cross-sectional implications for equally-talented managers.

[20] The concept of M-SHC sits naturally between the theoretical polar extremes of F-SHC and GHC (e.g., Becker, 1975). No talent is universally general, and as Lazear (2009) notes, it is not easy to identify important skills which are unambiguously firm-specific.

3.1 M-SHC made redundant, if hired cross-market.

Recall that parameter ϕ captures the relative concentrations of total human capital across the two markets. Consider the m-ranked manager from M1. She has talent equal to that of the ϕm-ranked manager from M2; $T_1(m) = T_2(\phi m)$. The whole of this talent can be applied when working in her home market M1, but *if hired cross-market to an M2 firm* the part representing her M-SHC would become redundant. Assume then she would have a remaining *effective* talent (her GHC) there equal only to the total talent of the θm-ranked manager from M2, for some $\theta \geq \phi$. She therefore has GHC of just $T_2(\theta m) = T_1\left(\frac{\theta}{\phi}m\right)$. The difference, $T_1(m) - T_2(\theta m) \geq 0$, is her M-SHC; the talent reduction associated with migration from M1 to M2.[21]

If $\theta = \phi$, then M1 managers' human capital is entirely GHC. But if $\theta > \phi$, then M1 managers employed at M2 firms effectively suffer a talent *downgrade.* The parameter θ is therefore positively related to the *degree of specificity of the human capital* throughout the talent-rich market; it also captures the relative availability of total talent in M2 managers versus GHC in M1 managers.[22]

3.2 Equilibrium in the unified market.

Unification of M1 and M2, firms and managers, implies an integrated single market which we denote $M0_\theta$.[23] In contrast to GL, our managers' talent has two components, and firms from different markets will *disagree* on the overall ranking of managers;[24] we cannot unambiguously order the managers in the unified talent pool, nor straightforwardly appeal to PAM (except in the special case of 100%-GHC in Section 3.3.1).

We notionally divide $M0_\theta$ into two constituent 'sub-markets', $M1_\theta$ and $M2_\theta$, containing the top

[21] M2 managers could also have an M-SHC component in their total human capital. For example, the ϕm-ranked M2 manager with talent $T_2(\phi m) \equiv T_1(m)$ might have *effective* talent of only $T_1(\mu m)$ (for some $\mu > 1$) if employed away by an M1 firm. However, this would not affect the analysis since there is no question of these managers migrating to the already talent-rich M1; they could not improve their wages by migration, even if all of their human capital were general, no M1 firm would want to hire them at their high wage.

[22] The independent $T_i(\cdot)$-functional forms assumed for total talent, and the definitions of scalar parameters ϕ and θ impose a further $T(\cdot)$-functional form on the GHC of former-M1 managers. This has the desirable feature that GHC is positively correlated with their total talent; their ranking relative to each other is unchanged from the perspective of M2 firms. A by-product of this modelling choice is that their M-SHC is increasing in m, however this does not drive our results which are all derived and hold cross-sectionally i.e., keeping the rank of manager or firm constant. This reflects our focus on θ as a parsimonious *market-wide* measure of how much of the talent in M1 comprises M-SHC. In Appendix B, for robustness, we address the implications of using an even more general functional form for GHC, equivalently a non-scalar θ.

[23] The n-ranked firm in $M0_\theta$ is the $\frac{1}{1+\lambda}n$-ranked firm in $M1_\theta$ or the $\frac{\lambda}{1+\lambda}n$-ranked firm in $M2_\theta$.

[24] M2 firms and M1 firms still agree on the effective talent ranking among M1 managers alone, and among M2 managers alone, they just disagree on the ranking in the unified talent pool.

former-M1 and former-M2 firms respectively. The top former-M1 managers and top former-M2 managers will be allocated across these two sub-markets in equilibrium. No former-M2 managers will optimally be hired by M1 firms, so M1$_\theta$ is the post-unification sub-market that comprises exactly the M1 firms and 'stay-at-home' former-M1 managers; M2$_\theta$ is the sub-market that comprises exactly the M2 firms, former-M2 managers, *and* any *migrating* former-M1 managers. In equilibrium, firms in one sub-market will unambiguously agree on the ranking of managers allocated *to* that sub-market, according to their *effective* talent *in* that sub-market, i.e., net of any redundant M-SHC. Specifically, denote by $T_{i\theta}(m)$ the effective talent of a manager allocated to the Mi_θ market, where arguments m are understood to be her ranking *within* her allocated Mi_θ.

Such a local talent ranking re-establishes the potential for PAM *within* that sub-market, but this is not sufficient for equilibrium. It is further necessary that there be equilibrium *between* M1$_\theta$ and M2$_\theta$, i.e., no firm from one sub-market would prefer to recruit a manager allocated to the other sub-market. Specifically, since a former-M1 manager may be hired in M1$_\theta$ or M2$_\theta$, her wage must be identical in either market, recognizing her loss of M-SHC if she is hired by an M2$_\theta$ firm.

The proportion of managers migrating from former-M1 firms to M2$_\theta$-firms could vary at every rank. Denote by the function $F(m) \leq 1$, the fraction of formerly m-ranked M1 managers who 'stay-at-home' in M1$_\theta$, then a fraction $1 - F(m)$ are hired cross-market to M2$_\theta$.

Formally, an equilibrium for M0$_\theta$ consists of

i) a function $F(m)$, which determines the allocation of former-M1 and former-M2 managers across sub-markets M1$_\theta$ and M2$_\theta$;

ii) two parallel wage functions $w_{i\theta}(T_{i\theta}(m))$, which specify the market wage in the Mi_θ sub-market, of an allocated manager of *effective* talent $T_{i\theta}(m)$ in that market;

iii) two assignment functions $M_{i\theta}(n)$ which determine the index (within Mi_θ) of the manager assigned to the n-ranked firm within Mi_θ;

iv) each firm choosing its manager optimally, and choosing from the managers allocated to its *own* sub-market Mi_θ, i.e., $M_{i\theta}(n) \in \arg\max_m \; S_{i\theta}(n) T_{i\theta}(m) - w_{i\theta}(T_{i\theta}(m))$, and no manager from the *other* sub-market would strictly dominate; and

v) market clearing, each firm in each sub-market recruits one and only one manager from that sub-market.

In (ii), the functional forms of $w_{i\theta}(m)$ and $T_{i\theta}(m)$ have *not* been specified. Indeed, since their arguments relate to the rank of the manager within the sub-market $\mathrm{M}i_\theta$, their functional forms will depend on the endogenous allocation of managers across the sub-markets in equilibrium, described by $F(m)$.

In (iv), we permit firms to have access to the *entire* manager talent pool, but for an equilibrium $F(m)$ we require they *optimally* hire managers allocated to their *own* sub-market, i.e., no manager from the other sub-market would strictly dominate. For equilibrium *between* $\mathrm{M1}_\theta$ and $\mathrm{M2}_\theta$, the 'no arbitrage' wage for *any* former-M1 manager must therefore be the same whether she is hired at home in $\mathrm{M1}_\theta$, or away in $\mathrm{M2}_\theta$.

3.3 Equilibrium cross-market hiring.

Consider the former m-ranked M1 manager who, if hired by an M2 firm, would rank alongside (i.e., share the same effective talent in $\mathrm{M2}_\theta$ as) the former θm-ranked M2 manager. At any former-M1 manager rank $x \leq m$, a fraction $F(x) \leq 1$ 'stay-at-home' in $\mathrm{M1}_\theta$, and a fraction $1 - F(x)$ are hired away to $\mathrm{M2}_\theta$. Then, of the first m former-M1 managers, a total number $H(m) = \int_0^m F(x)\,dx$ stay-at-home in $\mathrm{M1}_\theta$, and a total number $m - H(m)$ migrate to $\mathrm{M2}_\theta$.

Our former m-ranked M1 manager therefore either ranks $H(m)$ in $\mathrm{M1}_\theta$, or she ranks $\theta m + m - H(m)$ in $\mathrm{M2}_\theta$. For an equilibrium, she must be hired by either the $H(m)$-ranked firm in $\mathrm{M1}_\theta$, or the $\theta m + m - H(m)$-ranked firm in $\mathrm{M2}_\theta$; at *marginal* costs to the respective firms equal to her marginal benefit to those firms, given her effective talent in either market and that of the managers she competes against there; *and* at a 'no arbitrage' wage identical in either market, $w_{1\theta}(H(m)) = w_{2\theta}(\theta m + m - H(m))$. The cross-border market for talent is characterized as follows:

Proposition 1 . ***Equilibrium migration in the unified market.***

i) At every talent ranking of former-M1 managers, the fraction who stay-at-home with M1 firms is the constant

$$F_e(\theta) = \begin{cases} \frac{1+\theta}{1+\lambda\left(\frac{\theta}{\phi}\right)^{\beta}} < 1 & when \quad \theta \in [\phi, \bar{\theta}_e) \\ 1 & when \quad \theta \geq \bar{\theta}_e \end{cases} \qquad where \quad \bar{\theta}_e = \lambda \left(\frac{\lambda}{\phi}\right)^{\frac{\beta}{1-\beta}} \geq \lambda, \tag{6}$$

while a proportion $1-F_e(\theta)$ *are hired by M2 firms.*[25]

ii) $\frac{dF_e}{d\lambda} \leq 0$ *and* $\frac{dF_e}{d\phi} \geq 0,$

iii) $F_e(\phi) = \frac{1+\phi}{1+\lambda}$, *the 100%-GHC case; but* $\frac{dF_e}{d\theta}$ *can be positive or negative depending on* $\theta, \beta, \phi, \lambda,$ *as follows*

a) if $\phi \in \left[\tilde{\phi}_e, \lambda\right)$, *where* $\tilde{\phi}_e = \frac{\lambda\beta}{1+\lambda(1-\beta)}$, *then* $F_e(\theta)$ *is monotonically increasing,* $\frac{dF_e}{d\theta} \geq 0$ *for all* $\theta \in [\phi, \bar{\theta}_e)$,

b) otherwise, if $\phi \in \left(0, \tilde{\phi}_e\right)$, *then* $F_e(\theta)$ *is initially decreasing,* $\frac{dF_e}{d\theta}\Big|_{\theta=\phi} < 0$; *reaches a minimum for some* $\check{\theta} \in \left(\phi, \bar{\theta}_e\right)$; *and is then increasing,* $\frac{dF_e}{d\theta}\Big|_{\theta \geq \check{\theta}} \geq 0$.■

The *unique* equilibrium of the unified market with M-SHC is characterized by a proportion $1-F_e$ of former-M1 managers *at every talent rank*, migrating to M2 firms. This leaves M1 firms needing to employ lower-ranked M1 managers who are promoted, and relegates M2 managers to lower-ranked M2 firms. Since the competition and wage structure in $M1_\theta$ is determined by the stay-at-home fraction F_e, we contribute a closed-form expression as a function of M-SHC.[26]

The properties of $F_e(\theta)$ are illustrated in Figure 1. Notice $F_e(\lambda) < 1$, so that strictly $F_e < 1$ *even* for $\theta \in [\lambda, \bar{\theta}_e)$. This means migration is strictly positive *even* when θ is significantly higher than λ, i.e., even when M1 managers have (relative) *GHC* which is *lower* than the (relative) *total* human capital of M2 managers (each relative to firm size in their own market). For example, in the case $\phi = 1.5$ and $\lambda = 2$, migration is positive on the whole of the domain $\theta \in \left(\phi, \bar{\theta}_e\right) = (1.5, 3.5556)$. Cross-market hiring occurs nevertheless, due to the surfeit of *total* talent in the talent-rich market creating poor opportunities and low wages. This drives M1 managers to seek opportunities elsewhere in $M2_\theta$, and makes it attractive for firms in the talent-poor market to hire them – despite the loss of their significant M-SHC and hence their relatively small contribution to the destination market. This subtle *brain drain* contribution emphasizes that it is the condition $\phi < \lambda$ which *creates* potential cross-market hiring pressure from M1 to M2, but that $\theta > \lambda$ does not necessarily neutralize this pressure.

[25] *If* $\beta \approx \frac{2}{3}$, then $\bar{\theta}_e \approx \lambda \left(\frac{\lambda}{\phi}\right)^2$.

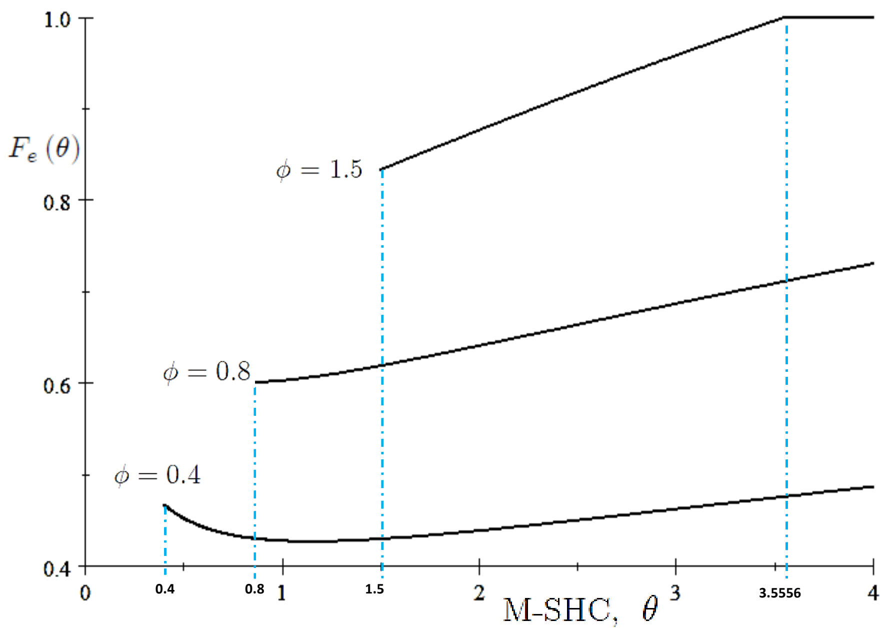
[26] In Appendix B we analyze a more general specification of GHC, wherein the former m-ranked M1 manager has GHC equivalent to the total talent of the former $t(m)$-ranked M2 manager. Equilibrium migration then varies by talent ranking, but is characterized less elegantly. Moreover, it is not clear in that set-up how to characterize the relative importance of M-SHC within *whole markets*; presently accomplished with our $t(m) \equiv \theta m$ specification.

Only when $\theta \geq \bar{\theta}_e$, is M-SHC so large that the prospective wage would be insufficient to make *any* migration worthwhile, and hence $F_e = 1$. This corresponds to standalone market wages $w_2(\theta m) \leq w_1(m)$, i.e., $w_2\left(\bar{\theta}_e m\right) = w_1(m)$, so there is no 'wage/talent arbitrage' incentive for cross-market hiring. *The two markets fail to integrate at all, even if all other barriers to integration are removed.* This possibility motivates our second interpretation of what may *initiate* integration. The first interpretation is when $\theta < \bar{\theta}_e$, and a grandiose *"Tear down this Wall!"* event removes a formal barrier between segregated markets. The second is 'organic integration' where no formal barriers exist, but initially $\theta \geq \bar{\theta}_e$ precludes integration until some shock or evolution brings θ *below* the $\bar{\theta}_e$ threshold. Such events could involve a reduction in θ, as some general skills increase in importance relative to market-specific skills; or even as some market-specific skills *become* more generally applicable. Further, (6) indicates that the 'no integration' threshold $\bar{\theta}_e$ for M-SHC is itself increasing in the relative size (value-sensitivity to talent) λ of the opportunity-rich market, and in the relative total talent of the talent-rich market (inversely proxied by ϕ), so that upward shocks to either may initiate organic integration. Irrespective of the route to integration, the squared factor (if $\beta \approx \frac{2}{3}$) in (6) implies that the no integration threshold increases rapidly with the relative talent-size disparity $\frac{\lambda}{\phi}$, with correspondingly *pessimistic implications for brain drain outcomes* over a broad range of $\theta \in \left(\lambda, \bar{\theta}_e\right)$.

Part (ii) shows that proportionally more managers migrate from M1 the greater the relative total talent disparity between the two markets.

Much more interestingly, part (iii) shows that F_e is *not* necessarily monotonic in θ. If ϕ is 'large enough' compared to λ (i.e., $\phi \in [\tilde{\phi}_e, \lambda]$), then the limited disparity in relative total talent between the markets means F_e *is* monotonically increasing in θ (for example, in Figure 1 with $\phi = 1.5$ and $\lambda = 2$). However, if M1 is sufficiently talent-rich (i.e., $\phi < \tilde{\phi}_e$), then F_e is U-shaped in θ (for example, in Figure 1 with $\phi = 0.4$ and $\lambda = 2$). Specifically, if ϕ is sufficiently low the talent-rich M1 segregated market has very high competition among managers, relatively low wages, and then F_e is initially *decreasing* in θ; *an increased proportion of M-SHC causes increased migration, even though it reduces the effective talent of migrating managers.* This is because migration is attractive up to the point where the increased competition in $M2_\theta$ (and reduced competition in $M1_\theta$) balances wages for M1 migrant *and* stay-at-home managers. Here, a higher θ softens the migration-induced increase of competition within $M2_\theta$; because they are downgraded further, the

Figure 1: Stay-at-home fraction $F_e(\theta)$, ($\lambda = 2$, $\beta = \frac{2}{3}$, so $\tilde{\phi}_e = 0.8$). Illustrated for three ϕ-values: $\phi = 1.5$ (so $\bar{\theta}_e\big|_{\phi=1.5} = 3.5556$); $\phi = \tilde{\phi}_e = 0.8$ (so $\bar{\theta}_e\big|_{\phi=0.8} = 12.5$, omitted for clarity); and $\phi = 0.4$ (so $\bar{\theta}_e\big|_{\phi=0.4} = 50$, omitted for clarity).



M1 migrants compete less with each other and with local M2 managers, permitting more migration. Even though the M1 migrants land at lower-ranked M2 firms, they still achieve a better wage than if θ was lower and they landed at a higher firm, but with more competition. This result is driven by the dual determinants of a manager's wage: first, her talent determines her ranking (which helps); second, the *density* of talent she competes against (which hurts). Eventually, of course, F_e is increasing in θ as fewer migrate when they suffer a sufficiently large talent discount, and none will migrate once $\theta \geq \bar{\theta}_e$.

In $M2_\theta$, competition is determined by the M1 migrating fraction $1 - F_e$, and we have:

Lemma 3 . ***Cross-market hiring by firms in the opportunity-rich market.***

At every firm-size rank, the fraction of M2 firms managed by former-M2 managers is $G_e = \frac{\theta}{\theta+1-F_e}$, *while the proportion* $1 - G_e$ *are managed by former-M1 managers.*

Here: $G_e < 1$ *for* $\theta \in [\phi, \bar{\theta}_e)$, *and* $G_e = 1$ *for* $\theta \geq \bar{\theta}_e$; $\frac{dG_e}{d\lambda} \leq 0$; $\frac{dG_e}{d\phi} \geq 0$; $G_e\big|_{\theta=\phi} = \frac{\phi}{\lambda} / \frac{1+\phi}{1+\lambda}$, *the 100%-GHC case; and* $\frac{dG_e}{d\theta} > 0$, *for all* $\theta \in [\phi, \bar{\theta}_e)$.■

F_e and G_e are distinct, but of course they both equal 1 if and only if $\theta \geq \bar{\theta}_e$. Notice the fraction G_e of top firms from M2$_\theta$ hiring cross-market is decreasing in M-SHC θ, *regardless* of whether F_e is monotonic. Even when the fraction migrating away from the talent-rich M1 market is increasing in M-SHC, more M-SHC means they pitch lower in the destination M2$_\theta$ market, are more dispersed there, and so form a smaller proportion of the local top manager population.

Remark 1: Having endogenously determined the *equilibrium* as being characterized by a *constant* fraction F_e of former-M1 managers at every rank 'staying-at-home', it is convenient here to note that for *any* constant stay-at-home fraction F, the m-ranked manager in M1$_\theta$ would formerly be $\frac{m}{F}$-ranked in M1, and the m-ranked manager in M2$_\theta$ would formerly be Gm-ranked in M2, (or $\frac{Gm}{\theta}$-ranked in M1), where $G = \frac{\theta}{\theta+1-F}$, so that talent rankings in either sub-market could be expressed

$$T_{1\theta}(m) = K - B_{1\theta}\frac{m^\beta}{\beta} \qquad \textit{where } B_{1\theta} = \frac{B_1}{F^\beta} \geq B_1, \tag{7}$$

$$T_{2\theta}(m) = K - B_{2\theta}\frac{m^\beta}{\beta} \qquad \textit{where } B_{2\theta} = G^\beta B_2 \leq B_2. \tag{8}$$

In particular, for the *equilibrium* fraction F_e, we have

Proposition 2 . *Unified market equilibrium; talent distribution and wages.*

i) The M1$_\theta$ sub-market associated with former-M1 firms has only stay-at-home former-M1 managers, distributed with talent $T_{1\theta e}(m) = K - B_{1\theta e}\frac{m^\beta}{\beta}$, where $B_{1\theta e} = \frac{B_1}{F_e^\beta} \geq B_1$.

ii) The M2$_\theta$ sub-market has former-M2 managers and some former-M1 managers, together distributed with effective talent $T_{2\theta e}(m) = K - B_{2\theta}\frac{m^\beta}{\beta}$, where $B_{2\theta e} = G_e^\beta B_2 \leq B_2$.

iii) Wages in the respective sub-markets are $w_{i\theta e}(m) = \frac{A_i B_{i\theta e}}{1-\beta} m^{-(1-\beta)}$.

iv) Wages at same-sized firms are related $w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = \left(\frac{\theta}{\phi}\right)^{\beta(1-\beta)} w_{1\theta e}\left(\frac{n}{1+\lambda}\right)$,
where $w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right)\Big|_{\theta=\phi} = w_{1\theta e}\left(\frac{n}{1+\lambda}\right)\Big|_{\theta=\phi}$, the 100%-GHC case, a single wage function;
$\frac{d}{d\theta}\left[w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right)/w_{1\theta e}\left(\frac{n}{1+\lambda}\right)\right] \geq 0$, the wage disparity is increasing in M-SHC; and
$w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right)/w_{1\theta e}\left(\frac{n}{1+\lambda}\right)\Big|_{\theta=\bar{\theta}_e} = \left(\frac{\lambda}{\phi}\right)^\beta$, the segregated markets case of Lemma 2.■

Parts (i) and (ii) formalize that within the unified equilibrium two parallel 'sub-markets' coexist with distinct talent profiles and wage functions, the direct result of cross-market hiring described by

Proposition 1. Conveniently, despite migration and redundancy of M-SHC, the equilibrium retains the $T(\cdot)$-functional form for distribution of effective talent in either sub-market – a consequence of F_e and G_e remaining constant at every rank.

M1 firms match only with former-M1 managers but top talent is scarcer than before, diluted by managers promoted from lower ranks in M1. M2 firms match *either* with former-M2 managers, *or* with former-M1 managers. Hence, there is unambiguously more talent at the top than before for M2 firms, even though the migrating M1 managers dissipate their M-SHC.

Empirically, there can still be *cross-sectional variation in wages* across same-sized firms from the former M1 and M2 markets, *even when talent migration is freely permitted and significant.* However, the GL 'reference-firm size' effect in segregated markets is eroded in integrated markets, and more so when talent is mostly GHC. Part (iv) implies the *opportunity-rich market maintains more of its proportional wage premium, the more of the human capital from the talent-rich market is market-specific,* and so redundant on migration. Only when $\theta = \phi$, the 100%-GHC case, is there a *single* wage function and no cross-sectional variation in wages of same-sized firms across sub-markets. This special case provides a benchmark for our results on M-SHC, and is also the only case which is immediately soluble using PAM, so we analyze it here first.

3.3.1 The 100%-GHC case, $\theta = \phi$, no M-SHC.

The subscript $i = 0$ identifies functions and parameters relating to the combined post-unification market M0 when $\theta = \phi$.

Quite conveniently, pooling the continuum of firms from M1 and M2 into a single market, and re-ranking them by size, gives a combined distribution of firm size which also follows a Zipf Law.[27] Even more conveniently, in the 100%-GHC case, effective talent ranking is unambiguous and *also* follows the spacing described by Extreme Value Theory.[28] To our knowledge, we are the first to combine distributions in this way, facilitating tractable contributions to our research question.

Proposition 3 . ***100%-GHC, unified market equilibrium; firm size, talent distribution, and wages.***

[27] $S_0(n) = S_1\left(\frac{n}{1+\lambda}\right) = S_2\left(\frac{\lambda n}{1+\lambda}\right).$

[28] $T_0(n) = T_1\left(\frac{n}{1+\phi}\right) = T_2\left(\frac{\phi n}{1+\phi}\right)$, and so $B_0 = B_1\left(\frac{1}{1+\phi}\right)^{\beta} = B_2\left(\frac{\phi}{1+\phi}\right)^{\beta}$

i) Firm size in M0 follows a Zipf distribution $S_0(n) = \frac{A_0}{n}$, *where* $A_0 = A_1 + A_2$.

ii) Talent in M0 is distributed $T_0(n) = K - B_0 \frac{n^\beta}{\beta}$, *where* $B_0 = B_1 \left(1 + \left(\frac{B_1}{B_2}\right)^{\frac{1}{\beta}}\right)^{-\beta}$.

iii) The n-ranked firm matches with the n-ranked manager (PAM), at wage $w_0(n) = \frac{A_0 B_0}{1-\beta} n^{-(1-\beta)}$.■

The unique wage function implies *no cross-sectional variation in wages across same-sized firms from different markets.*

3.3.2 Winners and losers from unification.

Returning to the general case $\theta \geq \phi$ with M-SHC, by identifying where firms and managers in the desegregated market $M0_\theta$ would have ranked and matched in their former segregated markets, we now proceed to identify which firms and managers gain or lose from unification.

The changed competitive environment causes interrelated wage and productivity effects. First, there is a resorting effect on wages; as managers reposition in the combined talent pool they match against the pooled firm-size distribution to decrease or increase the size of firm they match with. Second, a manager's revised marginal product impacts the wage function and the pay she receives. Similarly, firms increase or decrease their wage bill as the talent of the manager they employ changes and the wage function adjusts. For clarity in the presence of re-matching, we therefore distinguish between wages *earned* by particular managers (Lemma 4) and wages *paid* by particular firms (Lemmas 5 and 6; Proposition 4). Finally, unification has a productivity effect on *gross* firm value (i.e., before wages), whereby M2 firms hire better than before, and M1 firms hire worse.

Lemma 4 . ***Unified market; manager wage earned, winners and losers.***

i) The former m-ranked M1 manager improves her wage to $w_{1\theta e}(F_e m) = \frac{1}{F_e} w_1(m)$; *and* $\frac{d}{d\theta} w_{1\theta e}(F_e m)$ *has the opposite sign to* $\frac{dF_e}{d\theta}$, *i.e.,* $w_{1\theta e}(F_e m)$ *is increasing / decreasing / maximized in* θ *exactly when migration* $1 - F_e$ *is.*

ii) The former m-ranked M2 manager earns reduced wage, $w_{2\theta e}\left(\frac{m}{G_e}\right) = G_e w_2(m)$; *and* $\frac{d}{d\theta} w_{2\theta e}\left(\frac{m}{G_e}\right) > 0$, *i.e.,* $w_{2\theta e}\left(\frac{m}{G_e}\right)$ *is monotonically increasing in* θ.

iii) The former $\frac{1}{1+\phi}m$*-ranked M1 manager and* $\frac{\phi}{1+\phi}m$*-ranked M2 manager have the same total talent. Their weighted average overall wage gain* $\Delta M_{0\theta e}(m)$ *is increasing then decreas-*

ing in θ*, where:* $\Delta M_{0\theta e}|_{\theta=\phi} = 0$*, the 100%-GHC case;* $\Delta M_{0\theta e} > 0$ *for all* $\theta \in (\phi, \bar{\theta}_e)$*; and* $\Delta M_{0\theta e}|_{\theta=\bar{\theta}_e} = 0$*, the segregated markets case.*

iv) M2 managers decrease their firm size in $M2_\theta$ *to* $S_{2\theta e}\left(\frac{m}{G_e}\right) = G_e S_2(m)$*;*

M1 managers who remain in $M1_\theta$ *increase their firm size to* $S_{1\theta e}(F_e m) = \frac{1}{F_e} S_1(m)$*;*

Ex-M1 managers who migrate to $M2_\theta$ *increase or decrease firm size to* $S_{2\theta e}((1 - F_e + \theta) m)$*, which is monotonically decreasing in* θ*, i.e.,* $\frac{d}{d\theta} S_{2\theta e}((1 - F_e + \theta) m) < 0$*, where:*

$$S_{2\theta e}((1 - F_e + \theta) m)|_{\theta=\phi} = S_{1\theta e}(F_e m)|_{\theta=\phi} > S_1(m) \quad (9)$$

$$S_{2\theta e}((1 - F_e + \theta) m)|_{\theta\in[\lambda,\bar{\theta}_e)} < S_1(m). \quad (10)$$

Former-M1 managers move, in the unified market, either to a larger M1 firm at a higher wage, or to a (larger or smaller) M2 firm at the same higher wage. Former-M2 managers move to a smaller M2 firm at a lower wage. The overall average wage for managers with the same total human capital *increases* with unification and attains an interior maximum in M-SHC.

Part (i) shows that unification allows former-M1 managers who stay-at-home to multiply their wage by $\frac{1}{F_e} > 1$. The reduced competition engendered by talent exodus improves their wage function by a factor $\frac{1}{F_e^\beta}$ through the $B_{1\theta e}$ coefficient, and simultaneously promotes their individual ranking which increases their wage by a further factor $\frac{1}{F_e^{1-\beta}}$. Of course, identical wage improvements accrue to former-M1 managers who migrate to $M2_\theta$. Proposition 1 showed that F_e can be non-monotonic in θ, and this impacts $M1_\theta$ wages accordingly; when ϕ is 'small enough' $\left(\phi < \tilde{\phi}_e\right)$ relative to λ, then $w_{1\theta e}(F_e m)$ is hump-shaped, and an increase in M-SHC paradoxically initially *increases* migration, thereby *improving* wages for M1 managers. However, when $\phi \in [\tilde{\phi}_e, \lambda)$ the first-order intuition dominates – an increase in θ hurts the competitive position of M1-managers, and decreases their migration and wages.

Part (ii) shows how former-M2 managers see their wages impacted by the factor $G_e < 1$, the product of a G_e^β factor via the $B_{2\theta e}$ coefficient in the wage function, and a $G_e^{1-\beta}$ factor due to relegation within the $M2_\theta$ pecking-order. M2-managers are unambiguously better off (i.e., less worse off) when M1-managers have higher M-SHC and so compete less against them; G_e and $w_{2\theta e}\left(\frac{k}{G_e}\right)$ are each monotonically increasing in θ.

Part (iii) shows that the presence of M-SHC leads to a *gain* in average wages of top managers overall; if part of the human capital of M1 managers is M-SHC ($\theta > \phi$), then former-M1 managers gain more than equally-talented former-M2 managers lose from desegregation. *This average increases initially with M-SHC (and for all $\phi < \lambda$) such that there is a level of M-SHC which delivers a maximum average wage increase to managers,* before more M-SHC eventually decreases average wages down towards pre-unification levels. Again, this is due to the competition-softening effects of θ. Only in the cases where there is no integration $\left(\theta \geq \bar{\theta}_e\right)$ or where there is no M-SHC ($\theta = \phi$) do we find an unchanged overall average wage of equally-talented former-M1 and former-M2 managers.[29]

Part (iv) emphasizes that desegregation relegates former-M2 managers to a smaller firm within $M2_\theta$, and promotes stay-at-home former-M1 managers to a larger firm within $M1_\theta$. However, migrating former-M1 managers may match at a larger *or smaller* firm within $M2_\theta$, the destination firm-size decreasing in M-SHC. Specifically, in the 100%-GHC case (9), migrating M1 managers achieve the same increased firm-size as their stay-at-home peers. For higher levels of M-SHC, they are not promoted as high, and eventually (e.g., certainly once $\theta = \lambda$, and beyond, in (10)), they experience a decrease in firm-size, while still receiving the same 'arbitrage-free' wage increase as their stay-at-home peers. Manager's effective talent certainly decreases when hired cross-market; their firm-size may increase or decrease; so the value they create may increase *or decrease.* Indeed, certainly for $\theta \in [\lambda, \bar{\theta}_e)$, this corresponds to our 'brain drain' interpretation of self-interested migration causing loss of talent to home firms, while significant redundancy of M-SHC gives relatively small benefit to the cross-market hiring firms in the destination market.

Shifting to the firms' perspective, we now consider the n-ranked $M0_\theta$ firm, size $S_0(n)$: In turn, the former $\frac{n}{1+\lambda}$-ranked M1 firm; the former $\frac{\lambda n}{1+\lambda}$-ranked M2 firm; and their weighted average.

Lemma 5 . ***Firms from talent-rich market; effect on wages, talent, and net shareholder value.***

After unification, for all $\theta \in [\phi, \bar{\theta}_e)$, former-M1 firms pay a higher wage to a manager of

[29] This implies that the market wage in the 100%-GHC case of Proposition 3 (iii) is $w_0(n) = \frac{1}{1+\phi} w_1\left(\frac{n}{1+\phi}\right) + \frac{\phi}{1+\phi} w_2\left(\frac{\phi n}{1+\phi}\right)$, i.e., managers from M1 gain exactly what same-talented managers from M2 lose.

lower talent than before

$$w_{1\theta e}\left(\frac{n}{1+\lambda}\right) = \frac{1}{F_e^{\beta}}w_1\left(\frac{n}{1+\lambda}\right) > w_1\left(\frac{n}{1+\lambda}\right), \quad (11)$$

$$T_{1\theta e}\left(\frac{n}{1+\lambda}\right) = T_1\left(\frac{n}{F_e(1+\lambda)}\right) < T_1\left(\frac{n}{1+\lambda}\right), \quad (12)$$

and the M1 firm's gross value (and net shareholder value) decrease. Also:

$\frac{d}{d\theta}T_{1\theta e}\left(\frac{n}{1+\lambda}\right)$ *has the same sign as* $\frac{dF_e}{d\theta}$, *whereas* $\frac{d}{d\theta}w_{1\theta e}\left(\frac{n}{1+\lambda}\right)$ *has the opposite sign.*■

Firms from formerly talent-rich M1 *lose* from desegregation. More migration unambiguously hurts the firm, reduces the available stay-at-home talent pool, increases the competition for that talent, increasing the wage paid by a factor $\frac{1}{F_e^{\beta}} > 1$, while reducing the quality of talent hired and decreasing firm value.

If ϕ is sufficiently high (relative to λ) that $F_e(\theta)$ is monotonic decreasing in θ, then post-unification M1 firms are worst-off in the 100%-GHC case, and more M-SHC progressively mitigates their loss on unification. But if ϕ is sufficiently low $\left(\phi < \tilde{\phi}_e\right)$ that $F_e(\theta)$ is U-shaped in θ, then M1 firms are worst off at the interior, migration-maximizing level of M-SHC, namely $\check{\theta} = \arg\min F_e(\theta)$ from Proposition 1. This corresponds to the maximized wage paid, coincident with the minimized talent hired.

Lemma 6 . ***Firms from opportunity-rich market; effect on wages, talent, and net shareholder value.***

After unification, for all $\theta \in [\phi, \bar{\theta}_e)$, *former-M2 firms pay a lower wage to a manager of higher talent*

$$w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = G_e^{\beta}w_2\left(\frac{\lambda n}{1+\lambda}\right) < w_2\left(\frac{\lambda n}{1+\lambda}\right), \quad (13)$$

$$T_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = T_2\left(\frac{G_e\lambda n}{1+\lambda}\right) > T_2\left(\frac{\lambda n}{1+\lambda}\right), \quad (14)$$

and the M2 firm's gross value (and net shareholder value) increase.

Also, $\frac{d}{d\theta}w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) > 0$; *and* $\frac{d}{d\theta}T_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) < 0$, *i.e., wage paid is monotonically increasing in* θ*; and hired talent (and gross value, and net shareholder value) are monotonically decreasing in* θ.■

Firms from formerly talent-poor M2 *gain* from desegregation. The influx of cross-market talent unambiguously increases the available talent pool, decreases the competition for that talent, scaling down the wage paid by a factor $G_e^{\beta} < 1$, while increasing the quality of talent hired and increasing firm value. The higher the M-SHC in the talent-rich M1, the greater the fraction $G_e(\theta)$ of M2 firms managed by former-M2 managers, post-unification, and the *less* the M2 firms gain. When $\theta \geq \bar{\theta}_e$ is there no migration and no change from the pre-unification case.

Remark 2: For *any* constant stay-at-home fraction F, from Remark 1, the *overall weighted average gross value* of same-sized firms from both markets can be written

$$V_{0\theta}(n) = S_0(n)\left[\frac{1}{1+\lambda}T_1\left(\frac{1}{F}\frac{n}{1+\lambda}\right) + \frac{\lambda}{1+\lambda}T_2\left(\frac{G\lambda n}{1+\lambda}\right)\right], \tag{15}$$

where $G = \frac{\theta}{\theta+1-F}$. In particular, for the *equilibrium fraction* F_e, we have:

Proposition 4 . ***Same-sized firms; overall average effect on wages, productivity, and net shareholder value.***

In equilibrium, the n-ranked firm in the unified market $M0_\theta$ *pays an average wage of* $w_{0\theta e}(n)$*, has average gross value of* $V_{0\theta e}(n)$*, and has average shareholder value* $P_{0\theta e}(n) = V_{0\theta e}(n) - w_{0\theta e}(n)$*, where for all* $\phi < \lambda$*:*

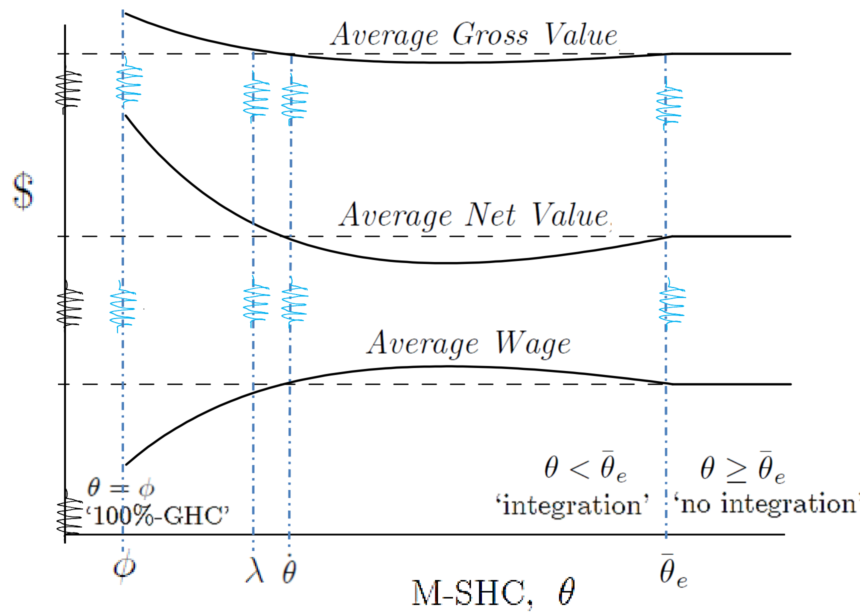
i) $w_{0\theta e}(n)$ *attains its global minimum value at* $\theta = \phi$ *(the 100%-GHC case,* $w_0(n)$*);* $w_{0\theta e}(n)$ *is increasing then decreasing in* $\theta \in \left(\phi, \bar{\theta}_e\right)$.

ii) $V_{0\theta e}(n)$ *and* $P_{0\theta e}(n)$ *attain their global maximum values at* $\theta = \phi$ *(the 100%-GHC case,* $V_0(n) = S_0(n)T_0(n)$ *); Each are decreasing then increasing in* $\theta \in \left(\phi, \bar{\theta}_e\right)$.■

The results of Proposition 4 are illustrated in Figure 2.

When $\theta = \phi$ (the 100%-GHC case), gross value, wage savings, and net shareholder value are each maximized. Gross value deriving directly from manager talent, firms from the talent-rich

Figure 2: Equilibrium effect of unification on average of same-sized firms across markets. Firm gross value $V_{0\theta e}$, wage $w_{0\theta e}$, and net shareholder value $P_{0\theta e} = V_{0\theta e} - w_{0\theta e}$, as functions of M-SHC θ.



market are losers from unification and firms from the talent-poor market are winners. Due to the complementarity of firm size and manager talent, this is *not* a zero-sum game. Relative to pre-unification there are gains from trade, realized through resorting and more efficient matching, unhindered by segregated markets and unmitigated by redundancy of M-SHC.

When $\theta \geq \bar{\theta}_e$, there is no migration and no change from the pre-unification case.

Between these polar extremes $\theta \in (\phi, \bar{\theta}_e)$, the effects of unification are non-monotonic in M-SHC for *all* $\phi < \lambda$. Moreover, if total human capital in the talent-rich market M1 is composed of sufficient M-SHC (versus GHC), $\theta \in \left(\dot{\theta}, \bar{\theta}_e\right)$, then *overall, on average, for the top firms across the two markets, desegregation of the market for managerial talent increases wages, decreases productivity, and destroys shareholder value.* It is worth emphasizing that these results pertain *regardless* of whether or not F_e happens to be monotonic in θ; they are *not* restricted to 'sufficiently low ϕ'.

Firms from the former talent-rich market M1 are still losers from unification and the firms from the talent-poor market are winners. In the 100%-GHC case the overall effect on same-sized firm average gross value is positive, and for low values of M-SHC this remains true. For higher values of M-SHC, while M1 managers still find it profitable to migrate, and M2 firms still find it profitable to hire them, the redundancy of substantial M-SHC limits the productivity gains to M2 firms. For same-sized firms, while M1 firms suffer the brain drain, and are forced to pay increased wages for reduced talent, the destination M2 firms' corresponding benefits (decreased wages for improved talent) are attenuated by the migrating managers' relegation far down the ranking, and consequently thinly-spread impact.[30]

Because gross value is U-shaped in θ, a *"Tear down this Wall!"* unification could actually cause a reduction in average productivity and shareholder value for top firms. Similarly, in an 'organic integration' scenario, as θ *decreases* from the 'no migration' threshold $\bar{\theta}_e$ so the average wage paid in a unified market increases, and productivity decreases. In the absence of any wall, and *in circumstances where skills formerly specific to market M1 become also more generally applicable to market M2* (i.e., a negative shock to θ), *subsequent market integration and cross-market hiring could destroy shareholder value.* Finally, even with no change in θ, value-destroying integration could be precipitated by an increase in $\bar{\theta}_e$, due either to an increase in the relative value-sensitivity of M2 firms to M1 firms, or to a increase in the relative total talent of M1 managers to M2 managers.

3.4 'Value-maximizing' constrained cross-market hiring.

Circumstances where free trade (in talent) may *reduce* overall productivity in equilibrium suggest some market failure. To explain this, the question naturally arises: what allocations of top talent (i.e., degree of cross-market hiring) would improve value?

Since the equilibrium outcome arises *endogenously* as a *fixed* stay-at-home fraction F_e of

[30] To see this mathematically, it is shown in the proof that $w_{0\theta}(n) = \left[\frac{1}{F_e^\beta} + \lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta\right]\frac{1}{1+\lambda}w_1\left(\frac{n}{1+\lambda}\right)$. The first $\frac{1}{F_e^\beta}$ term is just the increase multiple in M1-firm wages (from (11)), while the second $\lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta$ term comprises the λ-weight of more large M2-firms, the G_e^β decrease multiple in M2-firm wages (from (13)), and the $\left(\frac{\lambda}{\phi}\right)^\beta$ factor by which same-sized M2-firm wages originally exceeded those in M1 (from Lemma 2 (i)). As θ decreases just below $\bar{\theta}_e$ (where $F_e \approx 1$ and $G_e \approx 1$), changes in the first term dominate changes in the second, i.e., $\frac{d}{d\theta}\left[\frac{1}{F_e^\beta} + \lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta\right] = \beta\left[-\frac{1}{F_e^{\beta-1}}\frac{dF_e}{d\theta} + \lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^{\beta-1}\frac{dG_e}{d\theta}\right] < 0$ since $\frac{dF_e}{d\theta} > \lambda\left(\frac{\lambda}{\phi}\right)^\beta \frac{dG_e}{d\theta}$. The same analysis on the same factor gives the same result and intuition for gross value.

former-M1 managers at every rank, it is instructive to analyze the outcome for other fixed fractions, $F \neq F_e$. Of course, such arbitrary fractions F cannot be sustained as an equilibrium unless some extra constraint or regulator serves to enforce that F, and prohibit any further cross-market hiring and talent/wage arbitrage. Specifically, given any θ, any constant fraction F would impact the weighted average gross value $V_{0\theta}(n)$ as in (15) of Remark 2, and we have:

Proposition 5 . ***Constrained migration fraction, to maximize average productivity, wage savings, and net shareholder value.***

Given M-SHC parameter $\theta \geq \phi$,

i) the stay-at-home fraction at which average gross value $V_{0\theta}(n)$ is maximized; average wage paid $w_{0\theta}(n)$ is minimized; and average shareholder value $P_{0\theta}(n)$ is maximized; is

$$F_*(\theta) = \begin{cases} \frac{1+\theta}{1+\lambda\left(\frac{\theta}{\phi}\right)^{\frac{\beta}{1+\beta}}} < 1 & \text{when} \quad \theta \in [\phi, \bar{\theta}_*) \\ 1 & \text{when} \quad \theta \geq \bar{\theta}_* \end{cases} \qquad \text{where} \;\; \bar{\theta}_* = \lambda \left(\frac{\lambda}{\phi}\right)^{\beta} \tag{16}$$

ii) $F_*(\theta) > F_e(\theta)$ for $\theta \in \left(\phi, \bar{\theta}_e\right)$,

iii) $\bar{\theta}_* \in \left(\lambda, \bar{\theta}_e\right)$

iv) $F_*(\phi) = F_e(\phi) = \frac{1+\phi}{1+\lambda}$, in the 100%-GHC case. ■

If the objective of a social planner were to maximize $V_{0\theta}(n)$, constrained to manipulating fixed fraction F, then F_* would be the constrained first best; and simultaneously so for *all rankings, n.*[31] This closed-form expression for F_* serves as a benchmark for our market equilibrium outcome F_e.

Part (ii) emphasizes that the market equilibrium involves 'too much' migration compared to our constrained first best case: *Free trade in talent fails to maximize average gross value across same-size firms, and may even destroy it.* Part (iii) implies that $\bar{\theta}_* < \bar{\theta}_e$, so for $\theta \in [\bar{\theta}_*, \bar{\theta}_e)$ 'the market' permits positive migration, when ideally there would be none. Also $\bar{\theta}_* > \lambda$, so for

[31] If the objective were to maximize value for, say, the 'Top k' firms, and without regard to firms ranked below k, then the unconstrained first best $F(x)$ would generally *not* be a constant, but would depend on k. To see this, in the constant fraction case the former $\frac{k}{F(1+\lambda)}$-ranked M1 manager will match with the $\frac{k}{(1+\lambda)}$-ranked M1 firm (ranked k among all firms), but would match with M2 firms smaller than this. Hence, value for the 'Top k' firms could be improved by such managers *not* migrating to M2 firms.

$\theta \in [\lambda, \bar{\theta}_*)$ there is ideally *some* (i.e., constrained) cross-market hiring ($F_* (\theta) < 1$) even though M1 managers have (relative) *GHC* which is *lower* than the (relative) *total* human capital of M2 managers (each relative to firm size in their own market).

Finally, only in the 100%-GHC case of part (iv) does the market achieve the matching with the fullest potential for value creation.

4 Conclusion.

We ask how the distribution and composition of the human capital of the most talented managers in a market affects how (or whether) that market's most talent-sensitive firms and top managers compete with the firms and managers of another market, to develop a single (e.g., inter-industry, or cross-border) integrated market in talent. We vary parametrically the firm size *and* manager talent distributions across markets, clarifying cross-sectional implications. We then decompose talent into GHC and M-SHC constituents, and analyze the productivity, wage, and shareholder-value consequences when two distinct markets are unified and their talent pools desegregated.

Our analysis contributes a broad range of intuitively appealing results which attest to the robustness and adaptability of the competitive matching framework, while offering some new and challenging insights. We inherit the limitations of this approach. Our analysis is set in a frictionless competitive external market for managers, with no agency issues such as entrenchment (Bebchuk and Fried, 2003).[32] In identifying benefits and costs to unification, we abstract from other endogenous impacts on economic activity, product market competition, firm size (organic growth), M&A, etc. We do not model how managers accumulate human capital: our managers are mobile, but only after they have developed potentially the 'wrong' kind of M-SHC – we do not permit them to move mid-career to develop the 'right' kind. Our firms are presumed not to relocate to new markets simply to fully utilize talent (including M-SHC) in that market. Ours is not a general equilibrium or welfare analysis; our model applies only to the very largest firms and the very top managers, though these are often the ones that empirical researchers study, and whose lobbying influence may be strongest. Ours is not a dynamic model, it yields mere 'before and after' snapshots of wages and value.[33] Yet, the broad economic mechanisms and intuitions we analyze

[32] The GL framework also supports agency issues (Edmans, Gabaix and Landier, 2009; Edmans and Gabaix, 2011).
[33] Guren, Hémous and Olsen (2015) study shocks to an economy with Sector-SHC and overlapping generations.

are well-grounded and motivated by the transferability of human capital across national borders or industry sectors. Our model offers a framework to evaluate the possibility and consequences of integrating distinct talent markets, and to analyze the role of M-SHC versus GHC therein.

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Appendix A: Proofs of lemmas and propositions.

Lemma 1. (Gabaix and Landier, 2008). Wages in the market equilibrium.

A rigorous proof is contained in Gabaix and Landier (2008).■

Lemma 2. Segregated markets. Comparing same-sized firms, and equally-talented managers.

i) From size definitions, $S_2(\lambda n) = \frac{A_2}{\lambda n} = \frac{A_1}{n} = S_1(n)$. From talent definitions $T_1(n) = K - B_1\frac{n^\beta}{\beta}$ and $T_2(\lambda n) = K - B_2\frac{(\lambda n)^\beta}{\beta}$, and substituting $B_1 = B_2\phi^\beta$. From the wage functions, $w_2(\lambda n)/w_1(n) = \frac{A_2B_2}{(1-\beta)}(\lambda n)^{-(1-\beta)} \Big/ \frac{A_1B_1}{(1-\beta)}n^{-(1-\beta)} = \left(\frac{\lambda}{\phi}\right)^\beta$.

ii) Similarly, $S_2(\phi n) = \frac{A_2}{\phi n} = \frac{\lambda A_1}{\phi n} = \frac{\lambda}{\phi}S_1(n)$; $T_2(\phi n) = K - B_2\frac{(\phi n)^\beta}{\beta}$ and substituting $B_1 = B_2\phi^\beta$; $w_2(\phi n)/w_1(n) = \frac{A_2B_2}{(1-\beta)}(\phi n)^{-(1-\beta)} \Big/ \frac{A_1B_1}{(1-\beta)}n^{-(1-\beta)} = \frac{\lambda}{\phi}$; The inequalities follow directly by inspection.■

Proposition 1. Equilibrium migration in the unified market.

(i) Consider the former m-ranked M1 manager:

- Either, she ranks $H(m)$ in $M1_\theta$, where her talent is unchanged from before $T_{1\theta}(H(m)) = T_1(m) = K - \frac{B_1}{\beta}m^\beta$, so differentiating, and noting $H'(m) = F(m)$, implies that talent spacings satisfy

$$F(m)\,T'_{1\theta}(H(m)) = T'_1(m) = -B_1m^{\beta-1}. \tag{17}$$

The $H(n)$-ranked firm within $M1_\theta$ chooses its manager optimally, with first order condition $w'_{1\theta}(H(m)) = \frac{A_1}{H(n)}T'_{1\theta}(H(m))$, but for equilibrium *within* $M1_\theta$ (PAM) necessarily $H(m) = H(n)$, and substituting (17) we have

$$w'_{1\theta}(H(n)) = -\frac{A_1B_1}{H(n)\,F(n)}n^{\beta-1}. \tag{18}$$

- Otherwise, she ranks $\theta m + m - H(m)$ in $M2_\theta$, where her effective talent is reduced to her GHC, $T_{2\theta}(\theta m + m - H(m)) = T_2(\theta m) = K - \frac{B_2\theta^\beta}{\beta}m^\beta$, so that talent spacings satisfy

$$(\theta + 1 - F(m))\,T'_{2\theta}(\theta m + m - H(m)) = \theta T'_2(\theta m) = -B_2\theta^\beta m^{\beta-1}. \tag{19}$$

The $\theta n + n - H(n)$-ranked firm within $M2_\theta$ has first order condition $w'_{2\theta}(\theta m + m - H(m)) =$

$\frac{A_2}{\theta n+n-H(n)} T'_{2\theta}(\theta m + m - H(m))$ but for equilibrium within M2_θ necessarily $\theta n + n - H(n) = \theta m + m - H(m)$, and substituting (19) we have

$$w'_{2\theta}(\theta n + n - H(n)) = -\frac{A_2}{\theta n + n - H(n)} \frac{B_2 \theta^\beta n^{\beta-1}}{(\theta + 1 - F(n))}. \tag{20}$$

- Finally, the 'no arbitrage' equilibrium wage condition for any former-M1 manager requires she earns the same wage at home as she earns away $w_{1\theta}(H(m)) = w_{2\theta}(\theta m + m - H(m))$, so, differentiating,

$$F(m) w'_{1\theta}(H(m)) = (\theta + 1 - F(m)) w'_{2\theta}(\theta m + m - H(m)). \tag{21}$$

Substituting (18); (20); $A_2 = \lambda A_1$; $B_1 = B_2 \phi^\beta$; into (21), and solving gives

$$F(m) \frac{A_1 B_1}{H(m) F(m)} m^{\beta-1} = (\theta + 1 - F(m)) \frac{A_2}{\theta m + m - H(m)} \frac{B_2 \theta^\beta m^{\beta-1}}{(\theta + 1 - F(m))} \tag{22}$$

$$\theta m + m - H(m) = H(m) \frac{\lambda \theta^\beta}{\phi^\beta} \tag{23}$$

$$H(m) = \frac{1+\theta}{1 + \lambda \left(\frac{\theta}{\phi}\right)^\beta} m \tag{24}$$

and recalling $F(m) = H'(m)$ gives (6) which is *independent* of m.

Of course $F_e \leq 1$, so $F_e = 1$ when $\frac{1+\theta}{1+\lambda\left(\frac{\theta}{\phi}\right)^\beta} \geq 1$ i.e., $\theta \geq \lambda \left(\frac{\theta}{\phi}\right)^\beta$ which simplifies to $\theta^{1-\beta} \geq \lambda \left(\frac{1}{\phi}\right)^\beta$ i.e., $\theta \geq \lambda^{\frac{1}{1-\beta}} \left(\frac{1}{\phi}\right)^{\frac{\beta}{1-\beta}} = \lambda^{\frac{1}{1-\beta} - \frac{\beta}{1-\beta}} \left(\frac{\lambda}{\phi}\right)^{\frac{\beta}{1-\beta}} = \lambda \left(\frac{\lambda}{\phi}\right)^{\frac{\beta}{1-\beta}} = \bar{\theta}_e$.

Alternatively, notice that no manager will migrate if in the standalone markets $w_1(m) \geq w_2(\theta m)$, i.e., $\frac{A_1 B_1}{1-\beta} n^{-(1-\beta)} \geq \frac{A_2 B_2}{1-\beta} (\theta n)^{-(1-\beta)}$ which is $\phi^\beta \geq \lambda \theta^{-(1-\beta)}$ i.e., $\theta^{1-\beta} \geq \lambda \left(\frac{1}{\phi}\right)^\beta$ as above.

ii) By inspection of (6).

iii) Differentiating, $\frac{dF_e}{d\theta} = \frac{d}{d\theta}\left(\frac{1+\theta}{1+\lambda\left(\frac{\theta}{\phi}\right)^\beta}\right) = \frac{1+\lambda\left(\frac{\theta}{\phi}\right)^\beta\left(1-\beta\frac{1+\theta}{\theta}\right)}{\left(1+\lambda\left(\frac{\theta}{\phi}\right)^\beta\right)^2}$, so $\frac{dF_e}{d\theta}$ has the sign of its numerator $1 + \lambda\left(\frac{\theta}{\phi}\right)^\beta \left(1 - \beta\frac{1+\theta}{\theta}\right)$.

But $\frac{d}{d\theta}\left(1 + \lambda\left(\frac{\theta}{\phi}\right)^\beta \left(1 - \beta\frac{1+\theta}{\theta}\right)\right) = \beta(1-\beta)\lambda\frac{1+\theta}{\theta^2}\left(\frac{\theta}{\phi}\right)^\beta > 0$, so the numerator is increasing in $\theta \in [\phi, \bar{\theta}_e]$, and we know $F_e(\theta) < 1$ for $\theta < \bar{\theta}_e$ and $F_e(\bar{\theta}_e) = 1$, so $\frac{dF_e}{d\theta}\Big|_{\theta \nearrow \bar{\theta}_e} > 0$.

Evaluated at $\theta = \phi$, the numerator is $1 + \lambda \left(\frac{\theta}{\phi}\right)^{\beta} \left(1 - \beta \frac{1+\theta}{\theta}\right)\Big|_{\theta=\phi} = 1 + \lambda \left(1 - \beta \frac{1+\phi}{\phi}\right)$ which is negative if $\phi < \frac{\lambda\beta}{1+\lambda(1-\beta)} = \tilde{\phi}_e$, whereupon it is increasing in θ and eventually becomes positive.

Otherwise, (if $\phi \geq \tilde{\phi}_e$) then the numerator is always positive.

Also $F_e|_{\theta=\phi} = \frac{1+\phi}{1+\lambda} < 1$, and $F_e|_{\theta=\bar{\theta}_e} = 1$. So F_e is either decreasing then increasing (if $\phi < \tilde{\phi}_e$, whereupon $\frac{dF_e}{d\theta} = 0$ for some $\check{\theta} \in \left(\phi, \bar{\theta}_e\right)$), or monotonically increasing (if $\phi \geq \tilde{\phi}_e$). In fact F_e can be convex or concave in parts. ■

Lemma 3. Cross-market hiring by firms in the opportunity-rich market.

The formerly $\frac{1}{1+\theta}n$-ranked manager from M1 who migrates to M2 firms (a fraction $1 - F_e$ of $\frac{1}{1+\theta}n$-ranked M1 managers do so), ranks alongside the $\frac{\theta}{1+\theta}n$-ranked manager from M2 in the post-unification M2$_\theta$ sub-market where they together now rank $\frac{1-F_e}{1+\theta}n + \frac{\theta}{1+\theta}n = \frac{1+\theta-F_e}{1+\theta}n = \left(1 - \frac{F_e}{1+\theta}\right)n$. Hence $G_e = \frac{\frac{\theta}{1+\theta}n}{\left(1-\frac{F_e}{1+\theta}\right)n} = \frac{\theta}{1+\theta-F_e} = \frac{\theta}{1+\theta}\left(1 + \frac{1}{\lambda}\left(\frac{\phi}{\theta}\right)^{\beta}\right)$.

Of course $G_e \leq 1$, so $G_e = 1$ when $\frac{\theta}{1+\theta-F_e} \geq 1$ i.e., $F_e \geq 1$ i.e., $\theta \geq \bar{\theta}_e$. Also, by direct substitution, we can easily verify that $G_e|_{\theta=\bar{\theta}_e} = 1$.

Differentiating $\frac{dG_e}{d\theta} = \frac{d}{d\theta}\left(\frac{\theta}{1+\theta}\left(1 + \frac{1}{\lambda}\left(\frac{\phi}{\theta}\right)^{\beta}\right)\right) = \frac{1}{\lambda(1+\theta)^2}\left(\frac{\phi}{\theta}\right)^{\beta}\left((1-\beta) + \lambda\left(\frac{\theta}{\phi}\right)^{\beta} - \theta\beta\right)$, which is positive because $\theta \leq \lambda\left(\frac{\theta}{\phi}\right)^{\beta}$.■

Proposition 2. Unified market equilibrium; talent distribution and wages.

i) $T_{1\theta e}(m) = T_1\left(\frac{m}{F_e}\right) = K - B_1 \frac{\left(\frac{m}{F_e}\right)^{\beta}}{\beta} = K - B_{1\theta e}\frac{m^{\beta}}{\beta}$ where $B_{1\theta e} = \frac{B_1}{F_e^{\beta}} \geq B_1$.

ii) $T_{2\theta e}(m) = T_2(G_e m) = K - B_2 \frac{(G_e m)^{\beta}}{\beta} = K - B_{2\theta e}\frac{m^{\beta}}{\beta}$ where $B_{2\theta e} = G_e^{\beta} B_2 \leq B_2$.

iii) By Lemma 1.

iv) The n-ranked firm in the combined market M0$_\theta$ is either the former $\frac{n}{1+\lambda}$-ranked M1 firm in the M1$_\theta$ sub-market, or the former $\frac{\lambda n}{1+\lambda}$-ranked M2 firm in the M2$_\theta$ sub-market.

Dividing their respective wages $w_{2\theta e}\left(\frac{\lambda}{1+\lambda}n\right) / w_{1\theta e}\left(\frac{n}{1+\lambda}\right) = \frac{A_2 B_{2\theta e}}{1-\beta}\left(\frac{\lambda n}{1+\lambda}\right)^{-(1-\beta)} \Big/ \frac{A_1 B_{1\theta e}}{1-\beta}\left(\frac{n}{1+\lambda}\right)^{-(1-\beta)}$ $= \frac{B_{2\theta e}}{B_{1\theta e}}\lambda^{\beta} = \left(\frac{F_e G_e \lambda}{\phi}\right)^{\beta}$. Noting $F_e G_e = \frac{1+\theta}{1+\lambda\left(\frac{\theta}{\phi}\right)^{\beta}} \cdot \frac{\theta}{1+\theta}\left(1 + \frac{1}{\lambda}\left(\frac{\phi}{\theta}\right)^{\beta}\right) = \frac{\theta}{\lambda}\left(\frac{\phi}{\theta}\right)^{\beta}$ our ratio simplifies to $w_{2\theta e}\left(\frac{\lambda}{1+\lambda}n\right) / w_{1\theta e}\left(\frac{n}{1+\lambda}\right) = \left(\frac{\theta}{\lambda}\left(\frac{\phi}{\theta}\right)^{\beta}\frac{\lambda}{\phi}\right)^{\beta} = \left(\frac{\theta}{\phi}\right)^{\beta(1-\beta)}$.

Next $w_{2\theta e}\left(\frac{\lambda}{1+\lambda}n\right) / w_{1\theta e}\left(\frac{n}{1+\lambda}\right)\Big|_{\theta=\phi} = 1$;

$\frac{d}{d\theta}\left[w_{2\theta e}\left(\frac{\lambda}{1+\lambda}n\right) / w_{1\theta e}\left(\frac{n}{1+\lambda}\right)\right] = \frac{d}{d\theta}\left(\left(\frac{\theta}{\phi}\right)^{\beta(1-\beta)}\right) = \frac{\beta}{\phi}(1-\beta)\left(\frac{\theta}{\phi}\right)^{-\beta^2+\beta-1} \geq 0$;

and $w_{2\theta e}\left(\frac{\lambda}{1+\lambda}n\right) / w_{1\theta e}\left(\frac{n}{1+\lambda}\right)\Big|_{\theta=\bar{\theta}_e} = \left(\frac{\lambda}{\phi}\left(\frac{\lambda}{\phi}\right)^{\frac{\beta}{1-\beta}}\right)^{\beta(1-\beta)} = \left(\left(\frac{\lambda}{\phi}\right)^{\frac{1}{1-\beta}}\right)^{\beta(1-\beta)} = \left(\frac{\lambda}{\phi}\right)^{\beta}$.■

Proposition 3. 100%-GHC, unified market equilibrium; firm size, talent distribution, and wages.

i) The n-ranked firm in M0 is the $\frac{n}{1+\lambda}$-ranked firm in M1 or the $\frac{\lambda n}{1+\lambda}$-ranked firm in M2, so $S_0(n) = S_1\left(\frac{n}{1+\lambda}\right) = S_2\left(\frac{\lambda n}{1+\lambda}\right)$. Hence $S_0(n) = (1+\lambda)\frac{A_1}{n} = (1+\lambda)\frac{A_2}{\lambda n} = \frac{A_1+A_2}{n}$.

ii) The n-ranked manager in M0 is the $\frac{n}{1+\phi}$-ranked manager in M1 (or the $\frac{\phi n}{1+\phi}$-ranked manager in M2), so $T_0(n) = T_1\left(\frac{n}{1+\phi}\right)$ or $T_2\left(\frac{\phi n}{1+\phi}\right)$. Hence $B_0 = B_1\left(\frac{1}{1+\phi}\right)^{\beta}$ or $B_2\left(\frac{\phi}{1+\phi}\right)^{\beta}$. Eliminate ϕ by substituting $\phi = \left(\frac{B_1}{B_2}\right)^{\frac{1}{\beta}}$.

iii) Follows from the definitions, PAM, and Lemma 1.■

Lemma 4. Unified market; manager wage earned, winners and losers.

i) $w_{1\theta e}(F_e m) = \frac{A_1 B_{1\theta e}}{1-\beta}(F_e m)^{-(1-\beta)} = \frac{A_1 B_1 F_e^{-\beta}}{1-\beta}(F_e m)^{-(1-\beta)} = \frac{1}{F_e} w_1(m) \geq w_1(m)$.

where $\frac{d}{d\theta} w_{1\theta e}(F_e m) = w_1(m)\frac{d}{d\theta}\left(\frac{1}{F_e}\right) = -\frac{w_1(m)}{F_e^2}\frac{dF_e}{d\theta}$.

ii) $w_{2\theta e}\left(\frac{m}{G_e}\right) = \frac{A_2 B_{2\theta e}}{1-\beta}\left(\frac{m}{G_e}\right)^{-(1-\beta)} = \frac{A_2 B_2 G_e^{\beta}}{1-\beta}\left(\frac{m}{G_e}\right)^{-(1-\beta)} = G_e w_2(m) \leq w_2(m)$,

where $\frac{d}{d\theta} w_{2\theta e}\left(\frac{m}{G_e}\right) = w_2(m)\frac{dG_e}{d\theta} > 0$ from Proposition 1.

iii) The weighted average wage change is

$\frac{1}{1+\phi}\left[w_{1\theta e}\left(\frac{F_e}{1+\phi} m\right) - w_1\left(\frac{1}{1+\phi} m\right)\right] + \frac{\phi}{1+\phi}\left[w_{2\theta e}\left(\frac{\phi}{G_e(1+\phi)} m\right) - w_2\left(\frac{\phi}{1+\phi} m\right)\right]$, then substituting from parts (i) and (ii), and from Lemma 2 part (i), this simplifies to

$\frac{1}{1+\phi}\left[\frac{1}{F_e} + \lambda G_e - (1+\lambda)\right] w_1\left(\frac{1}{1+\phi} m\right)$.

The factor $\left[\frac{1}{F_e} + \lambda G_e - (1+\lambda)\right] = \left[\frac{1}{\frac{1+\theta}{1+\lambda\left(\frac{\theta}{\phi}\right)^{\beta}}} + \lambda\frac{\theta}{1+\theta}\left(1 + \frac{1}{\lambda}\left(\frac{\phi}{\theta}\right)^{\beta}\right) - (1+\lambda)\right]$ simplifies to $\frac{1}{1+\theta}\left(\lambda\left(\frac{\theta}{\phi}\right)^{\beta} + \theta\left(\frac{\phi}{\theta}\right)^{\beta} - (\lambda+\theta)\right) = \frac{1}{1+\theta}\left(\frac{\theta}{\phi}\right)^{-\beta}\left(\lambda\left(\frac{\theta}{\phi}\right)^{\beta} - \theta\right)\left(\left(\frac{\theta}{\phi}\right)^{\beta} - 1\right) \geq 0$ because $\theta \geq \phi$ and also $\theta \leq \lambda\left(\frac{\theta}{\phi}\right)^{\beta}$ when $\theta \leq \bar{\theta}_e$ (from the proof of Proposition 1.)

Evaluated at $\theta = \phi$, this is 0.

Evaluated at $\theta = \bar{\theta}_e$, this is 0, since $F_e = G_e = 1$ there (equivalently $\bar{\theta}_e = \lambda\left(\frac{\bar{\theta}_e}{\phi}\right)^{\beta}$).

Differentiating $\frac{d}{d\theta}\left[\frac{1}{F_e} + \lambda G_e - (1+\lambda)\right] = -\frac{1}{F_e^2}\frac{dF_e}{d\theta} + \lambda\frac{dG_e}{d\theta}$

$= \frac{1}{(1+\theta)^2}\left[-\left(1 + \lambda\left(\frac{\theta}{\phi}\right)^{\beta}\left((1-\beta) - \beta\theta^{-1}\right)\right) + \left((1-\beta) - \theta\beta\right)\left(\frac{\phi}{\theta}\right)^{\beta} + \lambda\right]$.

Evaluated at $\theta = \phi$, this is $\frac{d}{d\theta}\left(\frac{1}{F_e} + \lambda G_e\right)\Big|_{\theta=\phi} = \frac{\beta}{(1+\phi)\phi}(\lambda - \phi) > 0$ so wage is initially increasing in θ (for all $\phi < \lambda$).

Evaluated at $\theta = \bar{\theta}_e$, this is $\frac{d}{d\theta}\left(\frac{1}{F_e} + \lambda G_e\right)\Big|_{\theta=\lambda\left(\frac{\lambda}{\phi}\right)^{\frac{\beta}{1-\beta}}}$

$= -\frac{(1-\beta)}{(1+\theta)^2}\left[\left(1-\left(\frac{\lambda}{\phi}\right)^{\frac{-\beta}{1-\beta}}\right)+\lambda\left(\left(\frac{\lambda}{\phi}\right)^{\frac{\beta}{1-\beta}}-1\right)\right] < 0$, so wage is eventually decreasing in θ.

iv) Holding firm constant, firm-size is unchanged, i.e.,$S_{i\theta}(m) = S_i(m)$, while managers re-match:

First $S_{2\theta e}\left(\frac{m}{G_e}\right) = \frac{A_2}{m/G_e} = G_e S_2(m) < S_2(m)$. Similarly $S_{1\theta e}(F_e m) = \frac{1}{F_e} S_1(m) > S_1(m)$.

Next $S_{2\theta e}((1-F_e+\theta)m) = \frac{1}{1-F_e+\theta} S_2(m) = \frac{\lambda}{1-F_e+\theta} S_1(m) = \frac{\lambda}{\theta} G_e S_1(m)$.

Differentiating $\frac{d}{d\theta} S_{2\theta e}((1-F_e+\theta)m) = S_1(m)\frac{d}{d\theta}\left(\frac{\lambda G_e}{\theta}\right)$

$= S_1(m)\frac{d}{d\theta}\left(\frac{\lambda+\left(\frac{\phi}{\theta}\right)^\beta}{(1+\theta)}\right) = -\frac{(\theta+\beta+\theta\beta)\left(\frac{\phi}{\theta}\right)^\beta+\theta\lambda}{\theta(\theta+1)^2} S_1(m) < 0.$

Also, $S_{2\theta e}((1-F_e+\theta)m)|_{\theta=\phi} = \frac{\lambda}{\phi} S_1(m)\, G_e|_{\theta=\phi} = \frac{1+\lambda}{1+\phi} S_1(m)$ from Proposition 1;

$S_{2\theta e}((1-F_e+\theta)m)|_{\theta=\lambda} = S_1(m)\, G_e|_{\theta=\lambda} < S_1(m)$ ■

Lemma 5. Firms from talent-rich market; effect on wages, talent, and net shareholder value.

$w_{1\theta e}\left(\frac{n}{1+\lambda}\right) = \frac{A_1 B_{1\theta e}}{1-\beta}\left(\frac{n}{1+\lambda}\right)^{-(1-\beta)} = \frac{1}{F_e^\beta}\frac{A_1 B_1}{1-\beta}\left(\frac{n}{1+\lambda}\right)^{-(1-\beta)} = \frac{1}{F_e^\beta} w_1\left(\frac{n}{1+\lambda}\right) > w_1\left(\frac{n}{1+\lambda}\right)$,

$T_{1\theta e}\left(\frac{n}{1+\lambda}\right) = T_1\left(\frac{n}{F_e(1+\lambda)}\right) < T_1\left(\frac{n}{1+\lambda}\right)$, since the m-ranked manager in $M1_\theta$ is exactly the $\frac{m}{F_e}$-ranked manager in M1.

$\frac{d}{d\theta} w_{1\theta e}\left(\frac{n}{1+\lambda}\right) = w_1\left(\frac{n}{1+\lambda}\right)\frac{d}{d\theta}\left(\frac{1}{F_e^\beta}\right) = -\beta\left(\frac{1}{F_e^{\beta+1}}\right) w_1\left(\frac{n}{1+\lambda}\right)\frac{dF_e}{d\theta}$,

$\frac{d}{d\theta} T_{1\theta e}\left(\frac{n}{1+\lambda}\right) = \frac{d}{d\theta} T_1\left(\frac{n}{F_e(1+\lambda)}\right) = B_1\left(\frac{n}{F_e(1+\lambda)}\right)^{\beta-1}\left(\frac{n}{1+\lambda}\right)\frac{1}{F_e^2}\frac{dF_e}{d\theta}$.■

Lemma 6. Firms from opportunity-rich market; effect on wages, talent, and net shareholder value.

$w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = \frac{A_2 B_{2\theta e}}{1-\beta}\left(\frac{\lambda n}{1+\lambda}\right)^{-(1-\beta)} = G_e^\beta \frac{A_2 B_2}{1-\beta}\left(\frac{\lambda n}{1+\lambda}\right)^{-(1-\beta)} = G_e^\beta w_2\left(\frac{\lambda n}{1+\lambda}\right) < w_2\left(\frac{\lambda n}{1+\lambda}\right)$,

$T_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = T_2\left(\frac{G_e \lambda n}{1+\lambda}\right) > T_2\left(\frac{\lambda n}{1+\lambda}\right)$, since the former $G_e m$-ranked manager from M2 now ranks m in $M2_\theta$.

$\frac{d}{d\theta} w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = \beta G_e^{\beta-1} w_2\left(\frac{\lambda n}{1+\lambda}\right)\frac{dG_e}{d\theta}$; and $\frac{d}{d\theta} T_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = \frac{d}{d\theta} T_2\left(\frac{G_e \lambda n}{1+\lambda}\right) = -B_2\left(\frac{G_e \lambda n}{1+\lambda}\right)^{\beta-1}\frac{dG_e}{d\theta}$,

and the monotonicity results follow from Lemma 3.■

Proposition 4. Same-sized firms; overall average effect on wages, productivity, and net shareholder value.

Weighted average wage, $w_{0\theta e}(n) = \frac{1}{1+\lambda} w_{1\theta e}\left(\frac{n}{1+\lambda}\right) + \frac{\lambda}{1+\lambda} w_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right) = \left[\frac{1}{F_e^\beta} + \lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta\right]\frac{1}{1+\lambda} w_1\left(\frac{n}{1+\lambda}\right)$.

Evaluated at $\theta = \phi$, $w_{0\theta e}(n)|_{\theta=\phi} = w_0(n)$ i.e., the 100%-GHC case.

Evaluated at $\theta \to \bar{\theta}_e$, then $w_{i\theta e}(k) \to w_i(k)$ so $w_{0\theta e}(n)$ equals pre-unification average levels.

Differentiating $\frac{d}{d\theta} w_{0\theta e}(n) = \frac{1}{1+\lambda} w_1\left(\frac{n}{1+\lambda}\right)\frac{d}{d\theta}\left[\frac{1}{F_e^\beta} + \lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta\right]$, dropping the factor $\frac{1}{1+\lambda} w_1\left(\frac{n}{1+\lambda}\right)$.

$\frac{d}{d\theta}\left[\frac{1}{F_e^\beta}+\lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta\right]=\beta\left[-\frac{1}{F_e^{\beta+1}}\frac{dF_e}{d\theta}+\lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^{\beta-1}\frac{dG_e}{d\theta}\right]$

$=\beta\left[-\frac{1}{F_e^{\beta+1}}\frac{1+\lambda\left(\frac{\theta}{\phi}\right)^\beta\left(1-\beta\frac{1+\theta}{\theta}\right)}{\left(1+\lambda\left(\frac{\theta}{\phi}\right)^\beta\right)^2}+\lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^{\beta-1}\frac{1}{\lambda(1+\theta)^2}\left(\frac{\phi}{\theta}\right)^\beta\left((1-\beta)+\lambda\left(\frac{\theta}{\phi}\right)^\beta-\theta\beta\right)\right]$

Evaluated at $\theta=\phi$, this reduces to $\beta\frac{(1+\lambda)^\beta(\lambda-\phi)}{\phi(1+\phi)^{1+\beta}}>0$.

Evaluated as $\theta\to\bar{\theta}_e$, (so $F_e=G_e=1$ and $\bar{\theta}_e=\lambda\left(\frac{\bar{\theta}_e}{\phi}\right)^\beta$) this reduces to $-\frac{(1-\beta)\lambda\left(\bar{\theta}_e^\beta-\lambda^\beta\right)}{\bar{\theta}_e\left(1+\bar{\theta}_e\right)\phi^\beta}<0$, so $w_{0\theta e}(n)$ is maximised at some interior θ.

Similarly, average gross value, $V_{0\theta e}(n)=S_0(n)\left[\frac{1}{1+\lambda}T_{1\theta e}\left(\frac{n}{1+\lambda}\right)+\frac{\lambda}{1+\lambda}T_{2\theta e}\left(\frac{\lambda n}{1+\lambda}\right)\right]$

$=S_0(n)\left[\frac{1}{1+\lambda}T_1\left(\frac{n}{F_e(1+\lambda)}\right)+\frac{\lambda}{1+\lambda}T_2\left(\frac{G_e\lambda n}{1+\lambda}\right)\right]$

$=S_0(n)\left[K-\left(\frac{1}{1+\lambda}\frac{B_1}{\beta}\left(\frac{n}{F_e(1+\lambda)}\right)^\beta+\frac{\lambda}{1+\lambda}\frac{B_2}{\beta}\left(\frac{G_e\lambda n}{1+\lambda}\right)^\beta\right)\right]$

$=S_0(n)\left[K-n^\beta\left(\frac{1}{1+\lambda}\right)^{1+\beta}\frac{B_1}{\beta}\left[\frac{1}{F_e^\beta}+\lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta\right]\right]$, and the analysis proceeds as above on the factor $\left[\frac{1}{F_e^\beta}+\lambda\left(\frac{\lambda}{\phi}\right)^\beta G_e^\beta\right]$.■

Proof of Proposition 5. Constrained migration fraction, to maximize average productivity, wage savings, and net shareholder value.

i) Substituting $G=\frac{\theta}{(\theta+1-F)}$ and $B_1=B_2\phi^\beta$ into (15) gives

$V_{0\theta}(n)=S_0(n)\left[K-\frac{1}{\beta}\frac{1}{1+\lambda}\left(\frac{n}{1+\lambda}\right)^\beta B_1\left(\frac{1}{F^\beta}+\lambda\left(\frac{1}{\phi}\frac{\lambda\theta}{(\theta+1-F)}\right)^\beta\right)\right]$.

This is maximized when $\frac{1}{F^\beta}+\lambda\left(\frac{1}{\phi}\frac{\lambda\theta}{1+\theta-F}\right)^\beta$ is minimized.

First Order Condition $\frac{d}{dF}\left(\frac{1}{F^\beta}+\lambda\left(\frac{1}{\phi}\frac{\lambda\theta}{1+\theta-F}\right)^\beta\right)=\beta\left(\lambda\left(\frac{\lambda\theta}{\phi}\right)^\beta(\theta+1-F)^{-(1+\beta)}-F^{-(1+\beta)}\right)$

is zero when $\lambda\left(\frac{\lambda\theta}{\phi}\right)^\beta(\theta+1-F)^{-(1+\beta)}=F^{-(1+\beta)}$

i.e., $\left(\frac{\lambda F}{(\theta+1-F)}\right)^{1+\beta}\left(\frac{\theta}{\phi}\right)^\beta=1$ i.e., $\frac{\lambda F}{(\theta+1-F)}=\left(\frac{\phi}{\theta}\right)^{\frac{\beta}{1+\beta}}$ which gives F_*.

Second Order Condition, $\frac{d^2}{dF^2}\left(\frac{1}{F^\beta}+\lambda\left(\frac{1}{\phi}\frac{\lambda\theta}{1+\theta-F}\right)^\beta\right)=\beta(\beta+1)\left(\frac{1}{F^{\beta+2}}+\lambda\frac{\left(\theta\frac{\lambda}{\phi}\right)^\beta}{(1+\theta-F)^{\beta+2}}\right)>0$.

An identical analysis on the identical factor applies to $w_{0\theta}(n)=\frac{1}{1+\lambda}w_{1\theta}\left(\frac{n}{1+\lambda}\right)+\frac{\lambda}{1+\lambda}w_{2\theta}\left(\frac{\lambda n}{1+\lambda}\right)$

$=\frac{1}{1+\lambda}w_1\left(\frac{n}{1+\lambda}\right)\left[\frac{1}{F^\beta}+\lambda\left(\frac{\lambda\theta}{\phi(\theta+1-F)}\right)^\beta\right]$.

ii) By inspection of (6) and (16), noting they differ in the exponents $\frac{\beta}{1+\beta}<\beta$.

iii) and (iv) by inspection.■

Appendix B: Robustness to alternative specifications of GHC.

The m-ranked manager in the talent-rich (potentially talent-exporting) market M1, has total talent $T_1(m)$. To consider a more generalized division of this total into GHC and M-SHC components, let this manager have GHC of $T_2(t(m))$, such that if working 'away' for an M2 firm she would rank alongside the former $t(m)$-ranked M2 manager (our modelling approach has a scalar $t(m) \equiv \theta m$). Her M-SHC is then $T_1(m) - T_2(t(m))$. Presumably $t(m) \geq \phi m$, so that M-SHC is not negative. Also $t'(m) > 0$, so that more-talented managers from M1 have more GHC, otherwise we have the uninteresting outcome that M2 firms prefer strictly *lower*-ranked managers from M1. If $t(0) = 0$, then any-ranked firms in M2 can be managed by a former-M1 manager, but if $t(0) > 0$, then M2 firms ranked higher (i.e., smaller index) than $t(0)$ will be led exclusively by former-M2 managers and there exists a 'glass ceiling' in the M2 market above which M1 managers cannot rise; cross-market hiring will be relevant only below this ceiling.

The derivation of equilibrium migration then proceeds as in the proof of Proposition 1, except that our former m-ranked M1 manager either ranks $H(m)$ in $M1_\theta$, or she ranks $t(m)+m-H(m)$ in $M2_\theta$. In $M1_\theta$, expressions (17) and (18) are unchanged. However,

- in $M2_\theta$, her effective talent is reduced to $T_{2\theta}(t(m)+m-H(m)) = T_2(t(m)) = K - \frac{B_2}{\beta}(t(m))^\beta$, so that talent spacings satisfy

$$(t'(m)+1-F(m))\,T'_{2\theta}(t(m)+m-H(m)) = t'(m)\,T'_2(t(m)) = -B_2 t'(m)(t(m))^{\beta-1}, \tag{25}$$

  instead of expression (19). The $t(n)+n-H(n)$-ranked firm within $M2_\theta$ has first order condition $w'_{2\theta}(t(m)+m-H(m)) = \frac{A_2}{t(n)+n-H(n)} T'_{2\theta}(t(m)+m-H(m))$, but for equilibrium within $M2_\theta$ necessarily $t(n)+n-H(n) = t(m)+m-H(m)$, and substituting (25)

$$w'_{2\theta}(t(m)+m-H(m)) = -\frac{A_2}{t(m)+m-H(m)}\frac{B_2 t'(m)(t(m))^{\beta-1}}{(t'(m)+1-F(m))}, \tag{26}$$

  instead of expression (20).

- Finally, the 'no arbitrage' equilibrium wage condition is now $w_{1\theta}(H(m)) = w_{2\theta}(t(m)+m-H(m))$, so, differentiating,

$$F\left(m\right)w_{1\theta}'\left(H\left(m\right)\right)=\left(t'\left(m\right)+1-F\left(m\right)\right)w_{2\theta}'\left(t\left(m\right)+m-H\left(m\right)\right). \tag{27}$$

Substituting (18); (26); $A_2=\lambda A_1$; $B_1=B_2\phi^{\beta}$; into (27), and solving gives

$$H\left(m\right)=\frac{m+t\left(m\right)}{1+\lambda\phi^{-\beta}\left(\frac{t(m)}{m}\right)^{\beta-1}t'\left(m\right)}. \tag{28}$$

Recalling $F\left(m\right)=H'\left(m\right)$ gives

$$F\left(m\right)=\lambda\frac{\left[1+t'\left(m\right)\right]\left[\frac{\phi^{\beta}}{\lambda}\left(\frac{t(m)}{m}\right)^{1-\beta}+t'\left(m\right)\right]-\left[m+t\left(m\right)\right]\left[t''\left(m\right)-\frac{(1-\beta)t'(m)(mt'(m)-t(m))}{mt(m)}\right]}{\phi^{\beta}\left(\frac{t(m)}{m}\right)^{1-\beta}\left[1+\lambda\phi^{-\beta}\left(\frac{t(m)}{m}\right)^{\beta-1}t'\left(m\right)\right]^{2}} \tag{29}$$

which is no longer independent of m.

Of course if $t\left(m\right)\equiv\theta m$, then $F\left(m\right)=\frac{1+\theta}{1+\lambda\left(\frac{\theta}{\phi}\right)^{\beta}}=F_e\left(\theta\right)$ as in (6) of Proposition 1.